

Direction of Arrival Estimation for Shaped Dielectric Lens Array Antenna

¹S Ravishankar, ²Shushrutha K. S

Abstract

A cost effective smart antenna can be implemented by exciting the shaped dielectric lens by array antenna, by introducing dielectric lens the directivity is enhanced due to collimation of the rays in the specified direction, reduces interference and seals the array from environmental effects. However the major drawback with the use of a dielectric lens is compensation of the refractive effects of the lens in the receiving mode to obtain the angles of arrival to locate the desired signal. This paper dwells on the array signal processing algorithms to estimate the Direction of Arrival (DOA), using the algorithms MVDR and MUSIC. Analysis is continued for other conditions like resolution, different SNR values and presence of multiple sources with MUSIC and MVDR methods for 2x2 array antennas. Ray tracing method was introduced to analyze DOA in presence of dielectric lens. This lead to determination of virtual DOA due to refraction of rays in dielectric lens the shape of dielectric chosen is spheroid and 2x2 element linear arrays are chosen for analysis in this paper.

Keywords: Adaptive Beam Forming, Direction of Arrival, Minimum Variance Distortion less Response, Multiple Signal Classification.

Introduction

Adaptive array smart antenna involves the array signal processing to manipulate the signals induced on various antenna elements in such way that the main beam is directed towards the desired user and nulls are formed towards the interferers. This is achieved by two in-built properties of the smart antenna namely, adaptive beamforming and DOA. The DOA algorithms namely Bartlett, MVDR, MUSIC and Linear Prediction are described in Constantine A. Balanis and Panayiotis I. Ioannides [2], Lal Chand Godara [3], Jeffrey Foutz, Andreas Spanias and Mahesh K. Banavar [4], Harry L. Van Trees [5], Monson H. Hayes [6] and Frank Gross [8]. these authors provides a comprehensive study of the use of an antenna array to enhance the efficiency of mobile communication systems and provides details on the feasibility of antenna arrays for mobile communication applications.

A smart antenna also has to meet the contrasting requirement of compact form factor and high gain, to achieve this shaped dielectric lens can be used along with the array antenna elements. The use of dielectric lens antennas to enhance the performance of wireless broadband communication systems by

producing highly shaped beams is described by Carlos A. Fernandez et al. [11]. The subject of lens antenna design has been treated in the past by many authors [17, 18] in considerable detail. They point out the methods of design types of structure and general problems involved in the use of lenses.

In this paper a shaped dielectric lens antenna is excited by 2x2 array antenna by doing so high directivity, protection to array, cost of the system is greatly reduced. The major concern of this paper is to predict desired signal received in any direction. The complete analysis of this system to determine the DOA is carried out in two major steps firstly, Array antenna is analyzed to determine Direction Of Arrival (DOA) using two estimation methods, MVDR and MUSIC. Their pseudo spectrum equations are analyzed and simulated in Matlab considering resolution and varying SNR. Secondly, the array is now exciting lens antenna, analysis of this problem is carried out using ray tracing technique in which the shape of lens is first designed then the DOA of the signal is determined by fundamentals of ray theory. The paper presents the following studies:

- a. Comparison of various DOA algorithms using a linear array.

- b. To simulate the DOA algorithms viz. MUSIC and MVDR for array antenna with ellipsoidal lens combination.
- c. Compensate for the change in the DOA due to the refractive effect of the lens using Ray Tracing technique.

The remainder of the paper is organized as follows: Section II describes the received data model of the array, section III highlights the DOA algorithms used viz. MUSIC and MVDR, section IV focuses on ray tracing through the lens, and in section V conclusions are drawn out from previous simulated results of the corrected DOA based on the estimated virtual DOA after refraction through the lens for the case of a linear array is presented using the new technique.

Received Data Model

A uniform planar array considered on the yz plane of the coordinate system is shown in Fig. 1. The array is centered on the origin. There are M elements along any row in the y direction and N elements along any column in the z direction. Let d_y and d_z be the inter element spacing along y axis and z axis respectively. In figure 1, K signals arrive from K uncorrelated sources in K directions. Each received signal $x_k(t)$ includes additive white Gaussian noise. Under this model, the received signals can be expressed as a superposition of signals from all the sources and linearly added noise represented by [1, 3].

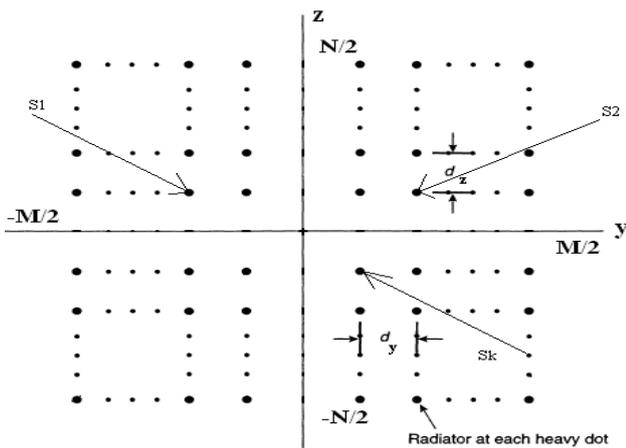


Figure 1: Uniform planar array

$$\mathbf{x}(t) = \sum_{k=1}^K \mathbf{a}(\theta_k, \phi_k) s_k(t) + \mathbf{n}(t) \tag{1}$$

where $s_k(t)$ is the incoming plane wave from the k^{th} source at time t and arriving from the direction $(\theta_k,$

$\phi_k)$ with θ_k is the elevation angle in the range $0^{\circ} \leq \theta_k \leq 180^{\circ}$ and ϕ_k is the azimuth angle, in the range $-90^{\circ} \leq \phi_k \leq 90^{\circ}$, $\mathbf{a}(\theta_k, \phi_k)$ is the array steering vector for the (θ_k, ϕ_k) direction of arrival, and $\mathbf{n}(t)$ represents additive white Gaussian noise. A single observation $\mathbf{x}(t)$ from the array is often referred to as a snapshot. Using a matrix notation (1) can also be written as [1,3]

$$\mathbf{x}(t) = \mathbf{A}(\Theta) \mathbf{s}(t) + \mathbf{n}(t) \tag{2}$$

Where $\mathbf{A}(\Theta)$ is the $(M*N) \times K$ matrix of array steering vectors. It is assumed that the arriving signals are uncorrelated and the number of arriving signals $K < (M*N)$. The received signal covariance matrix of size $(M*N) \times (M*N)$ is given by [1, 2],

$$\mathbf{R}_{xx} = \mathbf{A}(\Theta) \mathbf{R}_{ss} \mathbf{A}^H(\Theta) + \sigma_n^2 \mathbf{I} \tag{3}$$

Where σ_n^2 is the noise variance and \mathbf{I} is an identity matrix of size $(M*N) \times (M*N)$, \mathbf{R}_{ss} is the source signal covariance matrix.

Direction of Arrival Estimation Algorithms

The problem of localization of sources radiating energy by observing their signal received at the array antenna elements is of considerable importance occurring in many fields including radar, sonar, mobile communications, radio astronomy and seismology. In this section an estimation of the Direction Of Arrival (DOA) of narrowband sources of the same central frequency located in the far field of an array of antenna elements is considered and various DOA estimation methods are described. Data from an array of sensors are collected and the objective is to locate point sources assumed to be radiating energy that is detectable by the array elements.

Although most of the algorithms have been presented in the context of estimating a single angle per emitter (e.g. elevation only), generalizations to the elevation/azimuth case are relatively straight forward. Additional parameters such as frequency, polarization angle and range can also be incorporated provided that the response of the array is known as a function of these parameters.

Minimum Variance Distortionless Response Method

The MVDR method is similar to the delay-and-sum technique described, in that it measures the power of

the received signal in all possible directions. In this method, the output power is minimized with the constraint that the gain in the desired direction remains unity. Solving this constraint optimization problem for the weight vector we obtain [2]

$$W = \frac{R_{xx}^{-1}a(\theta)}{a^H(\theta)R_{xx}^{-1}a(\theta)} \quad (4)$$

Which gives the MVDR spectrum [2-7]

$$P_{MVDR} = W^H R_{xx} W = \frac{1}{a^H(\theta)R_{xx}^{-1}a(\theta)} \quad (5)$$

Again, the estimate of the true direction of arrival is the angle θ that corresponds to the peak value in this spectrum. The MVDR only requires an additional matrix inversion compared to the delay-and-sum method and exhibits greater resolution in most cases.

Music Algorithm

The received signal vectors form the received signal vector space whose vector dimension is equal to the number of array elements N . The received signal space can be separated into two parts: the signal subspace and the noise subspace. The signal subspace is the subspace spanned by the columns of $A(\Theta)$ [2, 22] and the subspace orthogonal to the signal subspace is known as the noise subspace. The subspace algorithms exploit this orthogonality to estimate the signals' DOAs. MUSIC has been the most widely examined. MUSIC stands for Multiple Signal Classification. The MUSIC algorithm was developed by Schmidt [22] by noting that the desired signal array response is orthogonal to the noise subspace. The signal and noise subspaces are first identified using Eigen decomposition of the received signal covariance matrix. Following, the MUSIC spatial spectrum is computed from which the DOAs are estimated. Inside the algorithm the general array manifold is defined to be the set

$$A = \{a(\theta_i) : \theta_i \in \Theta\} \quad (6)$$

The subspaces identified are typically achieved by Eigen decomposition of the auto covariance matrix of the received data R_{xx} . Using the model and assuming spatial whiteness, i.e., $E\{n(t)n^H(t)\} = \sigma^2 I$, the Eigen decomposition of R_{xx} will give the Eigen

values λ_n such that $\lambda_1 > \lambda_2 > \dots > \lambda_K > \lambda_{K+1} = \lambda_{K+2} = \dots = \lambda_N = \sigma_n^2$. Furthermore, it is easily shown that R_{xx} can be written in the following form [2, 4]

$$R_{xx} = \sum_{n=1}^N \lambda_n e_n e_n^H = E \Lambda E^H = E_s \Lambda_s E_s^H + E_n \Lambda_n E_n^H \quad (7)$$

$$R_{xx} = E_s \Lambda_s E_s^H + \sigma_n^2 E_n E_n^H = E_s \tilde{\Lambda}_s E_s^H + \sigma_n^2 I \quad (8)$$

where $E = [e_1, e_2, \dots, e_N]$, $E_s = [e_1, e_2, \dots, e_K]$, $E_n = [e_{K+1}, e_{K+2}, \dots, e_N]$, $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_N\}$, $\Lambda_s = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_K\}$, $\Lambda_n = \text{diag}\{\lambda_{K+1}, \lambda_{K+2}, \dots, \lambda_N\}$ and $\tilde{\Lambda}_s = \Lambda_s - \sigma_n^2 I$. The Eigen vectors $E = [E_s, E_n]$ can be assumed to form an orthonormal basis (i.e., $EE^H = E^H E = I$). The span of the K vectors E_s defines the signal subspace and the orthogonal complement spanned by E_n defines the noise subspace. For a detailed analysis of the Eigen structure properties of the signal auto covariance matrices R_{xx} and R_{ss} the reader is referred to [22]. Once the subspaces are determined the DOAs of the desired signals can be estimated by calculating the MUSIC spatial spectrum over the region of interest [2-7]

$$P_{MUSIC}(\theta) = \frac{a^H(\theta)a(\theta)}{a^H(\theta)E_n E_n^H a(\theta)} \quad (9)$$

Note that the $a(\theta)$ is the array response vectors calculated for all angles θ within the range of interest. Because the desired array response vectors $A(\Theta)$ are orthogonal to the noise subspace, the peaks in the MUSIC spatial spectrum represent the DOA estimates for the desired signals.

Pseudo-Spectrum Plots for MVDR and Music

1) Minimum Variance Distortionless Response method results

To estimate the true DOA, the MVDR method was executed for 2×2 -array antenna. To realize the resolving capability for varying SNR of MVDR algorithm, two sources are placed at $[-5^\circ \ 20^\circ]$ and $[5^\circ \ 25^\circ]$ in presence of AWG noise. The Pseudo spectrum given in Eqn.6 is considered for varying

SNR 5dB, 10dB, 15dB and 20dB in elevation angle is shown in Fig.4

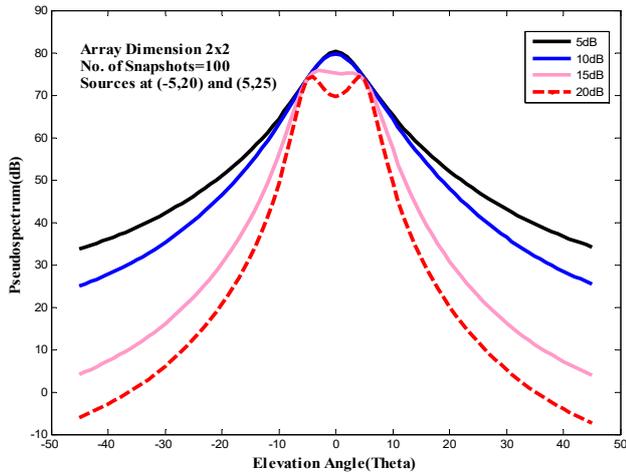


Figure 2: MVDR spectrum for closely placed sources for varying SNR

In Fig.2: Pseudo spectrum of MVDR algorithm is simulated for sources placed along elevation plane at -5° and 5° for varying SNR. It can be noticed at **5, 10 and 15dB** DOA angles are not resolved, at **20dB** two sources are resolved. Thus MVDR algorithm can resolve two sources placed 10° apart at **20dB** in presence of random noise.

The Pseudo spectrum of MVDR algorithm is simulated for two sources placed at angles $[-5^\circ 20^\circ]$ and $[5^\circ 25^\circ]$ with SNR 20dB for 2×2 array antenna. To realize the resolving capability of MVDR algorithm the Pseudo spectrum expression given in Eqn.6 is considered. The resolution capability of the algorithm in elevation angle two sources separated by $8^\circ, 10^\circ, 12^\circ$ and 14° is shown in Fig.3.

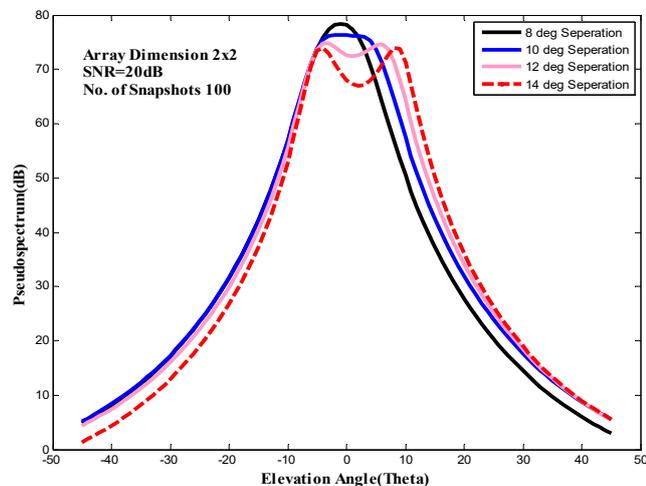


Figure 3: MVDR spectrum for closely placed sources

In Fig.3. The pseudo spectrum of MVDR algorithm is simulated for two closely separated sources. It can be noticed at 8° and 10° separation DOA angles are not resolved, at 12° separation two sources are almost resolved and for 14° separation the two sources are completely resolved as the peaks are predominant. Thus MVDR algorithm can resolve two sources placed 12° apart in presence of random noise.

2) MUSIC method results

To estimate the true DOA, the MUSIC method was executed for linear 2×2 array antenna. To realize the resolving capability for varying SNR of MUSIC algorithm, two sources are placed at $[5^\circ 20^\circ]$ and $[0^\circ 25^\circ]$ in presence of AWG noise. The Pseudo spectrum given in Eqn.9 is considered for varying SNR 5dB, 10dB, 15dB and 20dB in elevation angle is shown in Fig.4

In Fig.4 Pseudo spectrum of MUSIC algorithm is simulated for sources placed along elevation plane at -5° and 5° for varying SNR. It can be noticed at **0dB** and **5dB** DOA angles are not resolved, at **10dB** two sources can be identified and for **15dB** separation the two sources are completely resolved. Thus MUSIC algorithm can resolve two sources placed 10° apart at **10dB** in presence of random noise.

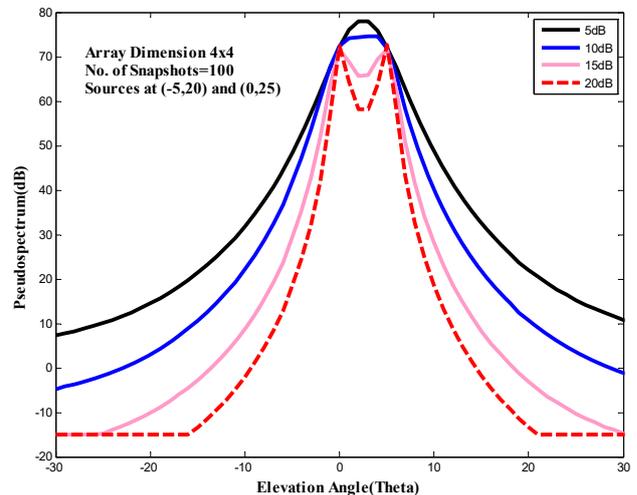


Figure 4: MUSIC spectrum for closely placed sources for varying SNR

To realize the resolving capability for varying SNR of MUSIC algorithm, two sources are placed at $[-5^\circ 20^\circ]$ and $[5^\circ 25^\circ]$ in presence of AWG noise. The Pseudo spectrum given in Eqn.9 is considered

for varying SNR **20dB**, **5dB**, **10dB** and **15dB** in elevation angle is shown in Fig.5

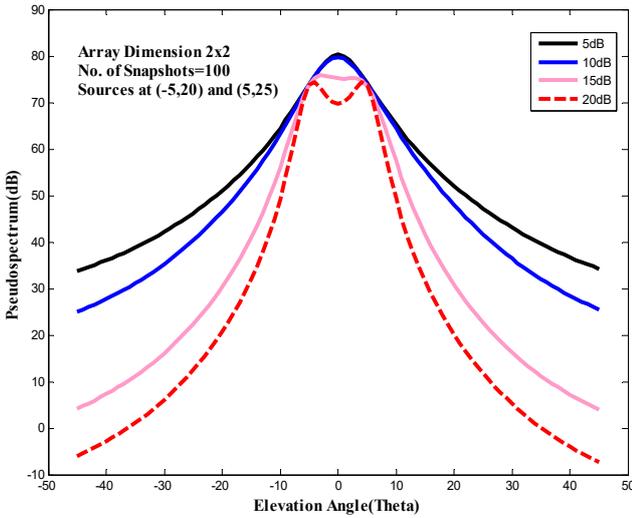


Figure 5: MUSIC spectrum for closely placed sources for varying SNR

In Fig.5: Pseudo spectrum of MUSIC algorithm is simulated for sources placed along elevation plane at -5° and 5° for varying SNR. It can be noticed at **5dB** and **10dB** DOA angles are not resolved, at **15dB** two sources can be identified and for **20dB** separation the two sources are completely resolved. Thus MUSIC algorithm can resolve two sources placed 10° apart at **10dB** in presence of random noise.

Ray Tracing through a Dielectric Lens

Figure 6 shows the cross section of a rotationally symmetric dielectric lens with its general contours S_1 and S_2 represented by (x_1, y_1) and (x_2, y_2) respectively [4]. The lens has dielectric constant ϵ_r . The distance between the origin and the first surface of the lens is f , called the focal length of the lens. Let T be the central thickness of the lens and D be the diameter of the lens aperture.

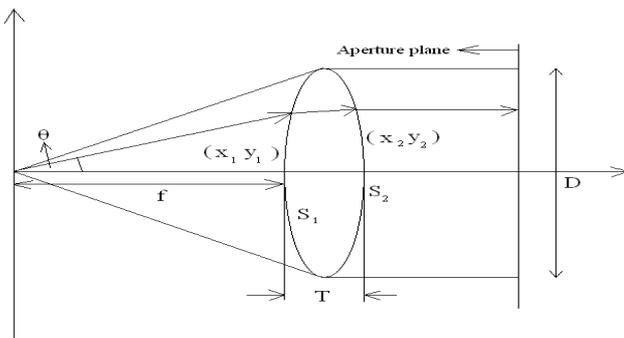


Figure 6: Geometry for dielectric lens design

The most important condition to be imposed in the derivation of the lens profiles is the path length constraint. Mathematically the path length condition is given by [4],

$$(x_1^2 + y_1^2)^{1/2} + n[(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2} - x_2 = (n - 1)T \tag{10}$$

Where the central ray has been used as a path length reference and $n = \sqrt{\epsilon_r}$ is the refractive index of the lens, which is greater than unity for real dielectrics. By taking the differential of y_1 with respect to x_1 in (10) the slope of the lens at (x_1, y_1) is obtained as [4],

$$\frac{dy_1}{dx_1} = \frac{nL_1(x_2 - x_1) - L_2x_1}{L_2y_1 - nL_1(y_2 - y_1)} \tag{11}$$

Where

$$L_1 = (x_1^2 + y_1^2)^{1/2}, L_2 = [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2} \tag{12}$$

Similarly, the slope of the contour at (x_2, y_2) is [4],

$$\frac{dy_2}{dx_2} = \frac{L_2 - n(x_2 - x_1)}{n(y_2 - y_1)} \tag{13}$$

In this paper we first examine a ellipsoidal lens lens with a flat surface on S_2 as show in figure 7. This lens transforms a spherical wavefront into a plane wave, or conversely focuses a beam from infinity onto focal point of the lens. The conditions imposed to derive S_1 are $x_2 = f + T$, the slope on S_2 being infinity, and the equal path length constraint. It can be readily shown that S_1 is a hyperbolic surface defined by [4],

$$y_2 = \left[\left[\frac{x + (n - 1)(F + T)}{n} \right]^2 - x^2 \right]^{1/2} \tag{14}$$

Ray Tracing Procedure

Backward Ray Tracing for Ellipsoidal Lens

- Step.1 Consider an incident ray with slope m , whose equation is given by $y_1 = mx_1 + c_1$.
- Step.2 Find the intersection point of incident ray at surface1 and apply Snell's law to obtain slope refracted ray.
- Step.3 Obtain the intersection point of refracted ray with the surface2.
- Step.4 Applying Snell's law, obtain the slope of final ray coming out of the lens and plot it till the point under consideration.

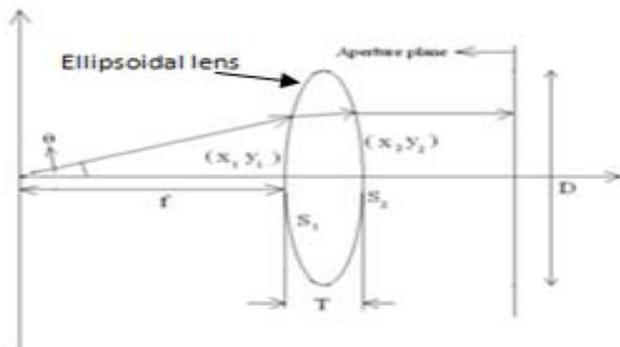


Figure 7: Ellipsoidal dielectric lens

Ray Tracing Simulation Outputs

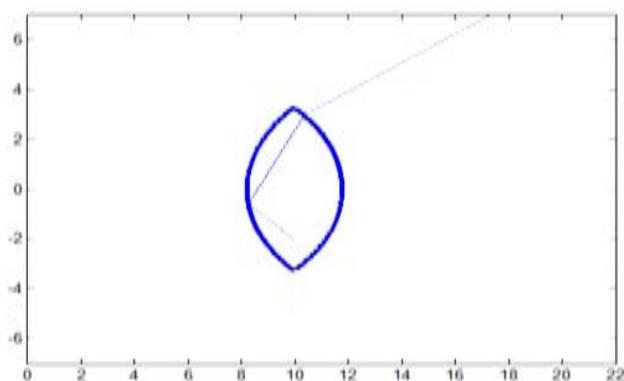


Figure 8: Ray tracing for 2X2 array antenna in presence of double ellipsoidal lens showing internal reflection

Internal reflection was observed in the ray tracing for 6-elements linear array antenna in presence of double ellipsoidal lens as shown in figure 8. Multiple internal refractions can also be plotted.

Relation between VDOA and ADOA

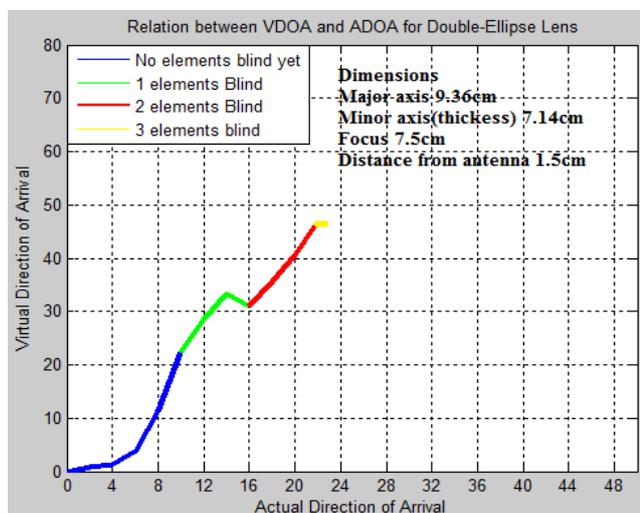


Figure 9: Relation between virtual DOA and actual

DOA for the Ellipsoid lens with 1.5cm separation distance between array and lens

Due to phenomenon called blind scan effect the performance of array degrades as elements start becoming blind. The relationship between Virtual DOA and Actual DOA is shown in Figure 10 with 1.5cm separation distance between lens and array and an operating frequency of 2.4 GHz.

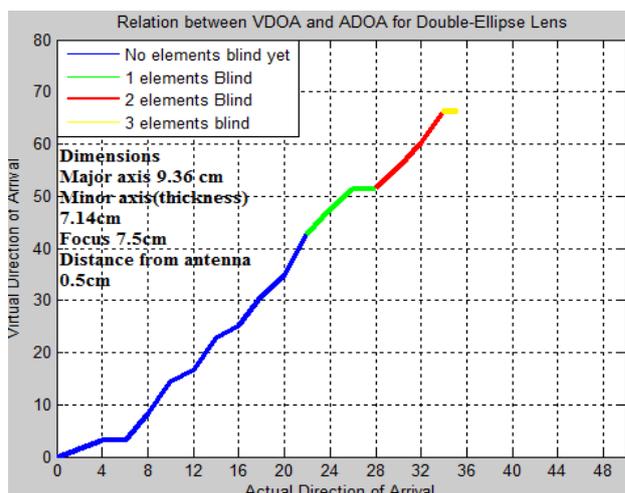


Figure 10: Relation between virtual DOA and actual DOA for the Ellipsoid lens with 0.5cm separation distance between array and lens

Due to phenomenon called blind scan effect the performance of array degrades as elements start becoming blind. The relationship between Virtual DOA and Actual DOA is shown in Figure 11 with 0.5cm separation distance between lens and array and an operating frequency of 2.4 GHz.

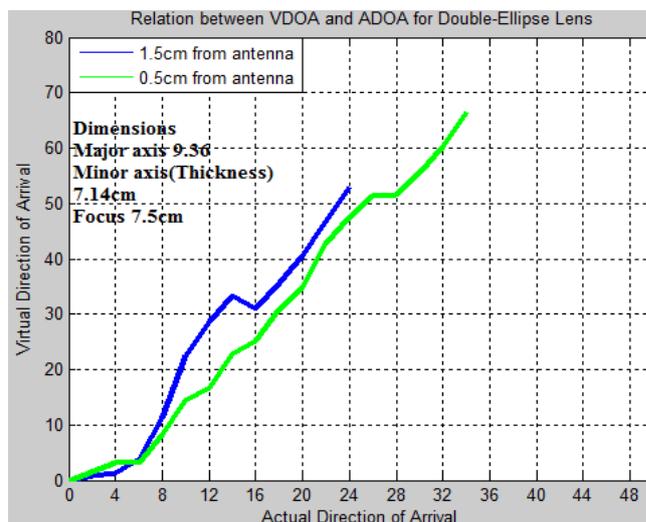


Figure 11: Comparison study of scan range for different separation distances between array and lens

The above figure shows the relation between Virtual DOA and Actual DOA for 1.5 cm and 0.5 cm separation distance between lens and array. It is found that for 0.5cm separation distance scan range is much higher than that of 1.5cm separation distance. Thus it can be concluded that reducing the separation distance between array and lens increases the scan range accordingly.

Conclusion

The objective of the paper is to simulate the different DOA algorithms viz. MVDR and MUSIC performance based on varying SNR and angular resolution without the presence of dielectric lens. Due to better performance, MUSIC and MVDR were used for 2X2 array with ellipsoidal dielectric lens combination, and compensate for the change in the DOA due to the refractive effect of the lens using ray tracing technique.

Recognizing that the smartness of an array antenna depends on the underlying spatial signal processing algorithms viz. DOA and ABF, a study of the DOA algorithms has been done initially without lens. The simulation results show that the performance of MVDR and MUSIC improves with increasing SNR of signals compared to DELAY-AND-SUM and Linear Prediction methods. MVDR and MUSIC showed lower rms error with increasing SNR, as compared to DELAY-AND-SUM and Linear Prediction methods.

The results of DOA estimation in two directions viz. elevation angle and azimuth angle using MUSIC and MVDR algorithms in the presence of shaped dielectric lens for 6-element linear array has been shown. Lens has the advantages like collimation of signals, inherent high bandwidth, and easiness in fabrication and cost effectiveness, which led the paper to adapt for the array antenna. The lens also causes refraction of the rays that result in the measured angle of arrival to be different (virtual DOA) from the actual one (correct DOA).

References

[1]. Constantine A. Balanis, "Antenna theory-Analysis and design", John Wiley & Sons, second edition, 2010, ISBN:9788126513932.
[2]. Constantine A. Balanis, Panayiotis I. Ioannides, "Introduction to Smart Antennas", Morgan & Claypool publishers, first edition, 2007, ISBN:1598291769.

[3]. Lal Chand Godara, "Smart Antennas", CRC press publication, 2004, ISBN:084931206X.
[4]. Jeffrey Foutz, Andreas Spanias, Mahesh K. Banavar, "Narrowband Direction of Arrival Estimation for Antenna Arrays", Morgan & Claypool publishers, 2008, ISBN:1598296507.
[5]. Harry L. Van Trees, "Optimum Array Processing-part 4 of Detection, Estimation and Modulation Theory", Wiley-Interscience publication, 2002, ISBN:0471093904.
[6]. Monson H. Hayes, "Stastical Digital Signal Processing and Modeling", John Wiley & Sons, 2009, ISBN:8126516100.
[7]. Samuel Silver, "Microwave Antenna Theory and Design", McGraw Hill publications, first edition, 1949, ISBN:0863410170.
[8]. Frank Gross, "Smart Antennas for Wireless Communications with MATLAB", McGraw-Hill publications, 2005, ISBN:007144789X.
[9]. Godara L.C, "Application of Antenna Arrays to Mobile Communications Part-I: Performance improvement, feasibility and system considerations", proceedings of the IEEE, July 1997, DOI:10.1109/5.611108, volume:85, issue:7, page(s):1031-1060.
[10]. Godara L.C, "Application of Antenna Arrays to Mobile Communications Part-II:Beam-forming and direction-of-arrival considerations", proceedings of the IEEE, August 1997, DOI:10.1109/5.622504, volume:85, issue:8, page(s):1195-1245.
[11]. Carlos A. Fernandes, "Shaped Dielectric Lenses for Wireless Millimeter-Wave Communications", Antennas and Propagation magazine IEEE, October 1999, DOI:10.1109/74.801527, volume:41, issue:5, page(s):141-150.
[12]. Yunxiang Zhang, Jian Wang, Zhiqin Zhao and Jianyu Yang, "Numerical Analysis of Dielectric Lens Antennas Using a Ray Tracing Method and HFSS Software", Antennas and Propagation magazine IEEE, August 2008, DOI:10.1109/MAP.2008.4653666, volume:50, issue:4, page(s):94-101.
[13]. Ravishankar.S and Prakash Biswagar, "Analysis of Dielectric Lens-Adaptive Array Antennas for Shaped Beam Applications", Sarnoff Symposium, proceedings of the IEEE, March 2006, DOI:10.1109/SARNOF.2006.4534746, page(s):1-4.
[14]. Y.T.Lo and S.W.Lee, "Antenna Handbook volume II Antenna Theory", Van Nostrand Reinhold publishers, 1993, ISBN:0442015933.

[15]. Richard C. Johnson and Henry Jasik, "Antenna Engineering Handbook", McGraw Hill publications, second edition, 1993, ISBN:007032381X.

[16]. Taguchi.M, Masuda.M, Shimoda.H and Tanaka.K, "Analysis of Arbitrarily Shaped Dielectric Lens Antenna", Antennas and Propagation Society International Symposium, proceedings of the IEEE, July 2001, DOI:10.1109/APS.2001.959837, volume:2, page(s):769-772.

[17]. Ravishankar S and Dharshak B.S, "Rapid Estimation of True Direction of Arrival for Dielectric lens based Adaptive Array Antennas", IEEE transactions on Signal Processing, Communications and Computing (ICSPCC), August 2012, DOI: 10.1109/ICSPCC.2012.6335678, page(s): 1-5.

[18]. Shushrutha K S, Ravishankar S Suleman Nadaf and Abdul Haq Nalband "Estimation of Actual Direction of Arrival for Ellipsoidal Dielectric Lens Array Antenna", IEEE International Microwave and RF Conference (IMaRC), 2014

[19]. D. G. Manolakis, Vinay K Ingle and Stephen M Kogon, "Statistical and adaptive signal processing spectral estimation, signal modeling, adaptive filtering and array processing", chapters 5, 6 and 10, McGraw Hill publications, 2000.

Author's Details

^{1,2}Department of Electronics and Communication, RV College of Engineering, Mysore Road, Bangalore, India

Email: shushruthks@gmail.com

Copy for Cite this Article- S Ravishankar and Shushrutha K.S, "Direction of arrival estimation for shaped dielectric lens array antenna", *International Journal of Science, Engineering and Technology*, Volume 4 Issue 2: 2016, pp. 449- 456.