

A Study of Fractal Dimension of Image

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Abstract

A Fractal Dimension (FD) deals with irregular shape of any object such as an image, it described as an index for characterizing fractal object by qualifying by the object's ratio of transformation as the complexity and in detail to transformation in respective scale. The fractal Dimension of image with irregular shape and can be representing the computation of FD via Euclidean dimension sets. There are different types of method of FD and have different application in different area like medical, human physiology, market trends, river networks, urban growth, turbulence etc. FD is widely used for image processing, image compression, image segmentation, image classification.

Index Terms: Fractal, Fractal Dimension, Image Processing, Image Compression, Image Segmentation, Image Classification.

Introduction

Fractal Dimension is now a days plays an important role in all field which deals with the objects (image) with an irregularity. The term "FRACTAL" was coined in the year of 1975 by Benoit Mandelbrot. It traces its origins to the Latin word "*fructus*", which means an irregular surface, like a broken stone. A fractal is an irregular shape which cannot be categorized as a known geometric shape and has some degree of non-regularity on all possible scales. Fractal are the kind of shape which can be seen in nature. For example, a right triangle can be described by Pythagorean theorem, but finding a regular right triangle in nature is difficult if not impossible. How can we define and determine the shape of a tree, mountain, cloud or a rock? How can the shape of a tumor be defined? IN fractal dimension world the fractal geometry is an attempt to find the answer to difficult question like these which arises with such these shapes of irregularity.

A fractal is a structure that is made up of similar forms and patterns that occur in many different sizes. The term fractal was first used by Benoit Mandelbrot to describes the repeating patterns that be observed occurring in many different structure [1]. For over one hundred years mathematicians have

made attempts to describe fractal shapes. Modern technology with ever increasing processing and imaging power of computers have made fractals re-popular as now they can be digitally rendered and explored. The use of algorithms to generate fractals produces complex visual pattern for image detections, reduction, expansion, compression, biomedical, medical, detecting natural occurrence of change etc. applications.

Fractal Dimension (FD) are the matrices that are useful in characterizing the geometry of sets that are too irregular to be explained in the more classical ways, which anyhow exhibit enough fractal regularity themselves. A Fractal Dimension holds information on the geometry of sets were traditional Euclidean measures may not hold enough information which are useful.

For example, a Koch curve (as shown in Figure 1) has zero area, infinite length and finite similarity dimension of $\log(4)/\log(3)$. A Koch curve is too irregular to be described well by Euclidean measures, its self-similarity allows for a simple description of its scaling properties via this Fractal Dimension. A less regular example is a random Koch curve which does not consist scaled images of itself, but rather a stochastic variation on these. The placement of the

details at any scale is always random and it can be highly variable in different implementation, however the rate at which the detail arises when the image is magnified is same as the original Koch curve. This can still be described by the same FD, and it exhibits "statistical self-similarity". With this example it is illustrated that "self-similar" and "fractal" are not synonyms to each other, statistical self-similarity or self-similarity is one more of fractal regularity.

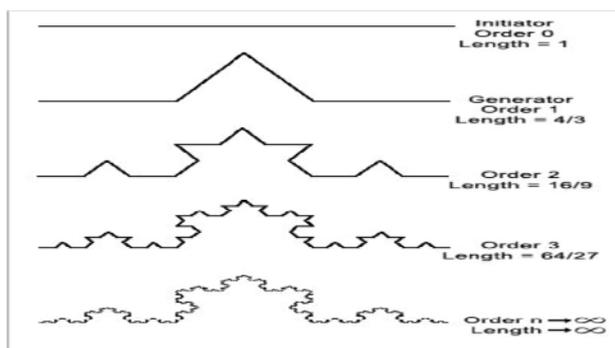


Fig. 1. Koch Curve method in Fractal Dimension

Construction of the Koch Curve follows the below steps.

Each interval is evenly divided into three sections.

The middle sections are replaced by the complementary sides of an equilateral triangle.

The above steps are repeated for every newly created interval, till the n^{th} iterations.

The Koch curve is the limit $n \rightarrow \infty$. The limit curve is further subdivided into four quarters, where each is an exact copy of the whole, such that it is scaled down by a factor of three. The curve is self-similar with a similarity dimension which is equal to $\log(4) / \log(3)$.

This Koch Curve is one of the methods in the Fractal Dimension and there are different methods are also available like Box Counting, Iterative function system (IFS), Positional Iterative function system (PIFS) etc, all the methods are for the different application.

The concepts of fractal dimension have been applied to a diverse set of natural phenomena. Objects

which are considered "natural fractals" cannot be described using classical geometry, to exhibit details which are meaningful over a large range of scales. They lend themselves to informative description using Fractal Dimension over finite range of scales. Objects are worth studying as fractals if they tend to show their details at too many scales and it becomes difficulty to be correctly approximated by classical geometric sets, these details can be consistently approximated with fractal models.

Literature Review

OSKAR CASTILLO 2002:

The author had introduced a new methodology for the estimation of fractal dimensions of a geometrical objects with the use of fuzzy logic techniques. To calculate the estimation of the fractal dimensions a numerical value using data as a time series for the specific problem was used for the specific problem. This numerical value gave an idea of the complexity of the time series or the geometrical object. There, however, was an underlying uncertainty in the estimation of the FD as we were using only a sample of the points of the object and the numerical algorithms are generally not very accurate for the calculations. Due to this reason, this proposed a new definition of the FD, that also incorporates the concept of a fuzzy set. This definition can be considered a more realistic but comparatively a weaker definition of the fractal dimension and it was named the "fuzzy fractal dimension".

Cadwen-Lun Zhong-KE, Fengjian-HU (2006):

An image classification method based on fractal dimension was presented Moreover, the fractal dimension of small complex traffic image was studied. The result of the experiment show fractal dimension be a good parameter in image classified and analysis. The classified result based on fractal dimension is the same as the result of classified according to person's subjective recent. To organize in a sequence of texture-image, texture-verge-image, texture-verge-smooth-image, verge-smooth-image and smooth-image, the corresponding fractal

dimension respectively is depending on the box counting measures.

Than D. Pham (2008):

Recent advances in biomedicine, pharmacology, and biotechnology open doors to the understanding how diseases are developed at the molecular and physiological level. This gain of understanding tremendously helps facilitate the design and discovery of drugs for therapeutic treatment. Despite the advances in the technology and new knowledge in systems biology, drug discovery is still a very low process without utilizing scientific computations that allow precise and rapid analysis of biological processes under trials. This paper particularly addresses fractals as a computational tool for analyzing molecular imaging data that appear to be very useful sources of information for understanding the interactions and behaviors of complex biological networks and the development of predictive medicine. We study herein some fractal characteristics of fluorescent microscope images of peroxisomes and propose the conceptual frameworks of fuzzy mixture fractal dimensions and fractal distortion measures for bioimage classification.

Sridharan B, Kepanna G, Thanush K (2013):

The performance of fractal-based coding algorithm such as standard fractal coding, quasi-lossless and improved quasi-lossless fractal coding has investigating their ability to compress magnetic resonance image (MRI) based on compression ratio, peak signal-to-noise ratio, encoding ratio. This all methods are useful in preserving the features of all the important features rich position of all the image like block and the remaining of the image by using fractal transformation. The most important task of fractal image compression of reduce of encoding time.

DI MARINO ET. AL. 2013:

This work was aimed on investigating the role that resolution plays in fractal dimension map

estimations, and on analyzing the role of the different surface spatial scales which are involved in the estimation process under consideration. The study was performed using a dataset of the Sky Med Synthetic Aperture Radar A.k.a. Sky Med SAR images and actual Cosmos which were relevant to the two different areas namely, the Italian city of Naples and the Bidi region of the country Burkina Faso. These images were acquired in the strip map and the enhanced spotlight modes. The behavior of the FD maps in the presence of areas which were having distinct characteristics with respect to the view-point of surface feature and land cover were discussed. In the presence of fine textural details significant differences among the estimated maps were obtained, this significantly affected the fractal dimension estimations for a high-resolution image in enhanced spotlight mode.

The result thus obtained showed that if interested to obtain a reliable estimation of the FD of the natural scene under observation strip map images should be the choice considering both computational and economic considerations. The combination of FD maps thus obtained from spotlight and strip-map images could be used to identify areas which are presenting a non-fractal behavior, for example an urban area. Along this guiding principle, a simple example of spotlight-strip map data fusion is present.

BO YANG 2016:

Dimensionally reduction techniques play a very essential role in signal processing, machine learning and data analytics. It is usually performed in a pre-processing stage, which is separate from data analysis, like classification or clustering. Finding representations which are of reduced dimension and more suited for the intentioned task is very appealing. This paper proposed a latent clustering framework and a joint factor analysis, aiming at learning low dimensional representations of tensor data and of matrix which are cluster-aware as well. The approach proposed here leverages tensor and matrix factorization models producing unique latent

representations of the data to unmask latent cluster structures, this would otherwise be obscured due to the freedom to apply an oblique transformation in the latent space. The latent cluster structure, at the same time, is used as prior information to enhance performance of the factorization.

Specific contributions included several problem formulations which were custom-built, various algorithms which were corresponding to the same and discussion of the associated convergence properties. In addition to extensive simulations, real world datasets like the MNIST image (Modified National Institute of Standards and Technology, which is a very large database of handwritten digits, commonly used in the training of various image processing systems.) and Reuters document data were employed to showcase the effectiveness of the approach which was proposed here.

DI MARINO ET. AL. 2017:

This work was aimed on investigating the role that resolution plays in fractal dimension map estimations, and on analyzing the role of the different surface spatial scales which are involved in the estimation process under consideration. The study was performed using a dataset of the Sky Med Synthetic Aperture Radar A.k.a. Sky Med SAR images and actual Cosmos which were relevant to the two different areas namely, the Italian city of Naples and the Bidi region of the country Burkina Faso. These images were acquired in the strip map and the enhanced spotlight modes. The behavior of the FD maps in the presence of areas which were having distinct characteristics with respect to the view-point of surface feature and land cover were discussed. In the presence of fine textural details significant differences among the estimated maps were obtained, this significantly affected the.

Nadia M.G.AL-saida, Aquee, H. Ali (2017):

Fractal image coding technique is considered as very effective for a higher compression ratio. Despite this method had received much attention because of its high resolution and fast decoding and many other

advantages, but it has not been used widely because it requires high computation time for the encoding process. This defect is considered as one of drawbacks of fractal image coding technique. However, many solutions have been proposed to facilitate the encoding time and optimize the computational time and resolution. In this paper, we proposed a new technique based on a fractal dimension that represents image complexity to be used as an index for the domain blocks in order to facilitate the searching process for each range block. In comparing to the original technique, the experiment shows that the encoding time is improved with a percentage of 30%, and the compromise between image quality and image compression is achieved.

Bignashwaran Bashara, Will Juice, Maheshwari R.V(2018):

Partial discharge (PD) measurement is an efficient method for condition monitoring of insulation in high-voltage (HV) power apparatus. Generally, phase-resolved PD (PRPD) patterns are commonly used to identify the PD sources. It is clearly recognized that there is a correlation between the PD patterns and the insulation quality. However, in the case of multiple PDs, the PRPD patterns partially overlapped in nature, which results in difficult to identify the types of partial discharges. In this proposed methodology, a combined algorithm of different edge detection methods with box-counting fractal image compression technique is used for fractal feature extraction. The extracted features used as the input vector for the classifiers for PD recognition. To evaluate the performance of the proposed methodology, artificially multiple PD sources are simulated in HV laboratory. The result of this proposed work shows better recognition for canny edge detected fractal features implemented with user define kernel multi-class nonlinear support vector machine which can be further used to assess the insulation properties for practical implementation in power industry.

Mohammed Omari and Salah Yaichi (2018)

In this wide area of Internet era and advances in telecommunications world has seen use of FD in the form of Fractal image processing in the form of image compression attract a great deal of research. Enhancements to the quality and ratio of image compression have been achieved through approaches such as neural networks and discrete transforms. However, other heuristic and bio-inspired methods, such as genetic algorithms, are still in the developmental stages. In this paper, we introduce a new image compression mechanism that exploits the relationship between rational numbers and their corresponding quotient representation. Each sub-image is mapped to a fractional number based on its RGB representation, and this fraction is then reduced to an efficient quotient. The appeal of using genetic algorithms is explained by the massive search needed to find a close fraction that can be reduced to a short quotient. We enhance the search by pre-calculating all possible rational numbers for a given set of numerators and denominators. While achieving a high compression ratio hence result in preventing the image quality.

Background

In the recent year the interest in applying fractals for image processing in different fields like compression, classification, encryption, etc has increased. Even though fractal compression has its limitations because of encoding complexity and information loss which degrade picture quality, currently the research community focuses on fractals owing to its advantageous properties such as fast decoding, resolution independence, high compression ratio and self-similarity.

Medical data size is found to be increasing day by day because of huge volume of digital data generated for analysis. Image compression has become an important issue for information storage and transmission. In many medical applications to ensure fast searching of images and to analyze measured data compression methods, image compression has become inevitable. Imaging techniques like magnetic resonance (MR),

computerized tomography and positron emission tomography are available. Yet, efficient compression methods are needed in many medical applications to ensure accurate analysis of stored data. A fractal is a structure that is made up of similar forms and patterns that occur in many different sizes.

With the use of different methodology of fractal compression like iterative frequent system methods like a novel quasi-lossless fractal coding algorithm are found to outperform standard fractal coding thereby proving the possibility of using fractal-based image compression algorithms for medical image compression. The proposed algorithm allows significant reduction of encoding time and improvement in the compression ratio.

Conclusion

Fractal Dimension have widely used in all field likes Medical, biomedical, geometrical field, etc. Fractal Dimension basically used for handling the object with irregularity in shape. Now a days FD has a wide range of application in every field. FD is mostly used or applied in the medical field or the telecommunication field to hide the important information from the others, this is done by image or text compression technique. There are many different methods to prevent the important things by encoding and decoding processes this is done by the Fractal coding method like FIS any also there are more different ways to do it. Encryption is also one of the important applications of FD.

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