



Radiation Effects on MHD Flow Past an Impulsively Started Infinite Isothermal Vertical Plate Using Finite Element Method

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Abstract- His study investigates the effects of thermal radiation on magnetohydrodynamic (MHD) free-convection flow past an impulsively started infinite isothermal vertical plate. The fluid is assumed to be viscous, incompressible, electrically conducting, and subjected to a uniform transverse magnetic field. The governing equations describing momentum and energy transport are formulated as coupled, nonlinear partial differential equations. Thermal radiation effects are incorporated into the energy equation using the Rosseland diffusion approximation.

Keywords- Magnetohydrodynamics (MHD), Thermal Radiation, Finite Element Method, Free Convection, Isothermal Vertical Plate, Heat Transfer, Boundary Layer Flow, Transient Analysis.

I. INTRODUCTION

Stokes [13] was the first to derive an exact solution to the Navier-Stokes equation for the case of flow past an impulsively started infinite horizontal plate in a viscous incompressible fluid. Soundalgekar [12] later on presented an exact solution for the flow past an infinite vertical isothermal plate impulsively started in a viscous incompressible fluid. Effects of free convection currents on the flow were studied. These investigations, however, are limited to the surrounding medium's typical temperature. Radiation impacts are significant when the surrounding fluid's temperature is high, and space technology is one instance of this. In such cases, one has to take into account the effects of radiation and free convection. In steady flows, such studies were presented by Cess [4], Arpacı [1], Cheng and Osakis [5], Hasegawa et al [7], Bankston et al [2], Hossain and Takhar [8], [9] and Hossain et al [10]. Raptis and Perdakis [11] solved the governing equation numerically and provided findings for flow via a uniformly accelerated vertical plate in the situation of unstable flows.

. Radiation effects on the natural convection flow past an impulsively stated infinite vertical plate were studied by Ganesan et al [6]. The Laplace-transform method is used to solve the converted dimensionless equations.

Here we propose to study thermal radiation effects on flow past an impulsively started infinite isothermal vertical plate in the presence of magnetic field. The dimensionless governing equations are solved using the Finite Element technique.



II. MATHEMATICAL FORMULATION

Here the unsteady flow of a viscous incompressible fluid past an impulsively started infinite isothermal vertical plate is considered. The y-axis is taken normal to the plate, whereas the x-axis is taken vertically upward along the plate.

It is also assumed that the radiation heat flux in the x-direction is negligible as compared to that in the y-direction. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. It is believed that the induced magnetic field and viscous dissipation are insignificant.

Under usual Boussinesq's approximation, the flow of a radiative fluid can be shown to be governed by the following system of equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + \nu \frac{\partial^2 u}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u \quad (1)$$

$$\rho c_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \quad (2)$$

where the Rosseland approximation (Brewster[3]) is used, which leads to

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial y'} \quad (3)$$

Where σ Stefan Boltzmann constant, k^* = mean absorption coefficient,
 ρ = density, k = thermal conductivity of fluid.

The initial and boundary conditions are as follows:

$$\begin{aligned} t' \leq 0: u = 0, T = T_\infty \text{ for all } y' \\ t' \geq 0: u = u_0, T = T_\infty + (T_w - T_\infty)At' \text{ at } y' = 0 \end{aligned} \quad (4)$$

$$u = 0, T \rightarrow T_w \text{ as } y' \rightarrow \infty$$

Where $A = \frac{u_0^2}{\nu}$ (constant), u_0 = velocity of plate, ν = kinematic viscosity.

It is assumed here that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature this is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms thus
 $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$ (5)

using equations (1.1.3) and (1.1.5) equation (1.1.2) gives

$$\rho c_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y'^2} + \frac{16\sigma T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y'^2} \quad (6)$$

on introducing the following non-dimensional quantities

$$\begin{aligned} U = \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad y = \frac{y' u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\ P_r = \frac{\mu c_p}{k} \text{ (Prandtl number)}, \quad R = \frac{k^* k}{4\sigma T_\infty^3} \text{ (Thermal radiation)} \end{aligned} \quad (7)$$

$$G_r = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3} \text{ (Grashof number)}, \quad M = \frac{\sigma B_0^2 \nu}{\rho u_0^2} \text{ (magnetic field parameter)}.$$



in equations (1.1.1) to (1.1.6), we obtain

$$\frac{\partial u}{\partial t} = G_r \theta + \frac{\partial^2 u}{\partial y^2} - Mu, \quad (8)$$

$$3RP_r \frac{\partial \theta}{\partial t} = (3R + 4) \frac{\partial^2 \theta}{\partial y^2}, \quad (9)$$

the initial and boundary conditions in non-dimensional form are

$$\begin{aligned} u = 0, \theta = 0 & \text{ for all } y, t \leq 0 \\ t > 0; u=1, \theta=1 & \text{ at } y=0 \\ u = 0, \theta = 0 & \text{ for all } y \rightarrow \infty \end{aligned} \quad (10)$$

III. METHOD OF SOLUTION

The Finite Element Technique (Ritz method) is used to solve equations (8) and (9) subject to initial and boundary condition given in equation (10). To obtain the difference equations, the region is divided into a grid or mesh of lines parallel to y and t axis. Solution of the difference equations is obtained at the intersection of these mesh lines, called nodes. The values of the dependent variables u and T at the nodal points along the planes $y=0$ are given by $u(0, t)$, $T(0, t)$ and thus are known from the boundary conditions. In the figure (1) h, k are the constant mesh sizes along y and t directions respectively. A scheme is required to find single value at next time level in terms of known values from present time level. We use finite element method to get Crank-Nicolson discretization.

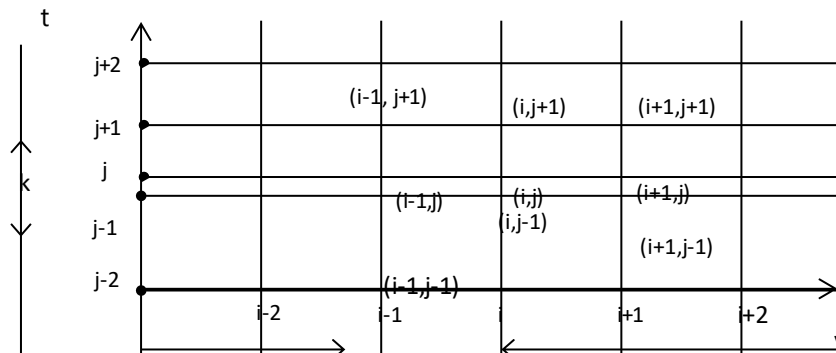


Figure (1.)

Consider equation (9)

$$\begin{aligned} 3RP_r \frac{\partial \theta}{\partial t} &= (3R + 4) \frac{\partial^2 \theta}{\partial y^2} \\ \Rightarrow \frac{\partial \theta}{\partial t} &= \frac{(3R + 4)}{3R} \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} \end{aligned}$$

Using finite element method with crank-nicolson discretization taking $h=0.1, k=0.01$ therefore $r = k / h^2 = 1$.

The element equation for the typical element (e) $y_j \leq y \leq y_k$ for the boundary value problem may be written as

$$j^e = 1/2 \int_{y_j}^{y_k} \left[\left(\frac{\partial \theta^e}{\partial y} \right)^2 + 2\theta^e P_r \frac{3R}{(3R + 4)} \frac{\partial \theta^e}{\partial t} \right] dy$$

For the linear piecewise approximate solution



$$\theta^e = N_j(y)\theta_j(t) + N_k(y)\theta_k(t)$$

the element equation is given by

$$j^e = \int_{y_j}^{y_k} \left\{ \begin{matrix} [N'_j & N'_k] \\ \theta_j \\ \theta_k \end{matrix} \right\} + 2P_r \frac{3R}{(3R+4)} [N_j \quad N_k] \begin{matrix} \theta_j \\ \theta_k \end{matrix} \right\} dy$$

$$\frac{\partial j^e}{\partial \theta_j} = \int_{y_j}^{y_k} \left\{ 2 \begin{matrix} [N'_j & N'_k] \\ \theta_j \\ \theta_k \end{matrix} \right\} N'_j + 2P_r \frac{3R}{(3R+4)} N_j [N_j \quad N_k] \begin{matrix} \theta_j \\ \theta_k \end{matrix} \right\} dy$$

$$\frac{\partial j^e}{\partial \theta_k} = \int_{y_j}^{y_k} \left\{ 2 \begin{matrix} [N'_j & N'_k] \\ \theta_j \\ \theta_k \end{matrix} \right\} N'_k + 2P_r \frac{3R}{(3R+4)} N_k [N_j \quad N_k] \begin{matrix} \theta_j \\ \theta_k \end{matrix} \right\} dy$$

$$\frac{\partial j^e}{\partial \theta_j} = \int_{y_j}^{y_k} \left\{ 2 \begin{matrix} [N'_j & N'_k] \\ \theta_j \\ \theta_k \end{matrix} \right\} N'_j + 2P_r \frac{3R}{(3R+4)} N_j [N_j \quad N_k] \begin{matrix} \theta_j \\ \theta_k \end{matrix} \right\} dy$$

$$\frac{\partial j^e}{\partial \theta_k} = \int_{y_j}^{y_k} \left\{ 2 \begin{matrix} [N'_j & N'_k] \\ \theta_j \\ \theta_k \end{matrix} \right\} N'_k + 2P_r \frac{3R}{(3R+4)} N_k [N_j \quad N_k] \begin{matrix} \theta_j \\ \theta_k \end{matrix} \right\} dy$$

$$\frac{\partial j^e}{\partial \phi^e} = \int_{y_j}^{y_k} \left\{ \begin{matrix} N'_j [N'_j & N'_k] \\ N'_k [N'_j & N'_k] \\ \theta_j \\ \theta_k \end{matrix} \right\} + P_r \frac{3R}{(3R+4)} \begin{matrix} N_j [N_j & N_k] \\ N_k [N_j & N_k] \\ \theta_j \\ \theta_k \end{matrix} \right\} dy$$

$$\frac{\partial j^e}{\partial \phi^e} = \int_{y_j}^{y_k} \left\{ \begin{matrix} [N'_j & N'_k] \\ [N'_j & N'_k] \\ N'_j [N'_j & N'_k] \\ N'_k [N'_j & N'_k] \\ \theta_j \\ \theta_k \end{matrix} \right\} + P_r \frac{3R}{(3R+4)} \begin{matrix} [N_j & N_k] \\ [N_j & N_k] \\ N_j [N_j & N_k] \\ N_k [N_j & N_k] \\ \theta_j \\ \theta_k \end{matrix} \right\} dy$$

Where prime denotes differentiation w.r.t. to 'y' and dot represent differentiation w.r.t. to 't'

Here $N'_j = \frac{-1}{h}$; $N'_k = \frac{1}{h}$; where $h = y_k - y_j$;

Simplifying, we get

$$\frac{(3R+4)}{3R} \frac{1}{P} \frac{1}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \theta_j \\ \theta_k \end{bmatrix} + \frac{h}{6} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} \theta_j \\ \theta_k \end{bmatrix} = 0$$

The condition for the extremization of the functional j^e w.r.t the nodal values

T_j and T_k are given by

$$\frac{\partial j^e}{\partial \phi^e} = \sum_{e=1}^{n+1} \frac{\partial j^e}{\partial \phi^e} = 0$$

In order to get the differential equation at the knot y_i we write the element equations for the elements



$$y_{i-4} \leq y \leq y_{i-3}; y_{i-3} \leq y \leq y_{i-2}; y_{i-2} \leq y \leq y_{i-1}; y_{i-1} \leq y \leq y_i;$$

$$y_i \leq y \leq y_{i+1}; y_{i+1} \leq y \leq y_{i+2}; y_{i+2} \leq y \leq y_{i+3}; y_{i+3} \leq y \leq y_{i+4};$$

For the elements $y_{i-4} \leq y \leq y_{i-3}$ and $y_{i-3} \leq y \leq y_{i-2}$ the assembled equation is given by

$$\frac{1}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2+2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} \theta_{i-4} \\ \theta_{i-3} \\ \theta_{i-2} \end{bmatrix} + \frac{(3R+4)}{3R} \frac{1}{P_r} \frac{1}{h^2} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+1 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta_{i-4} \\ \theta_{i-3} \\ \theta_{i-2} \end{bmatrix} = 0$$

Thus, assembling all the elements we get

$$\frac{1}{6} \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} \theta_{i-4} \\ \theta_{i-3} \\ \theta_{i-2} \\ \theta_{i-1} \\ \theta_i \\ \theta_{i+1} \\ \theta_{i+2} \\ \theta_{i+3} \\ \theta_{i+4} \end{bmatrix} + \frac{(3R+4)}{3R} \frac{1}{P_r} \frac{1}{h^2} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} \theta_{i-4} \\ \theta_{i-3} \\ \theta_{i-2} \\ \theta_{i-1} \\ \theta_i \\ \theta_{i+1} \\ \theta_{i+2} \\ \theta_{i+3} \\ \theta_{i+4} \end{bmatrix} = 0$$

We put the row corresponding to the knot 'i' to zero and write the



semi- discretization as

$$\frac{1}{6} \begin{pmatrix} \theta_{i-1} + 4\theta_i + \theta_{i+1} \\ \theta_{i-1} - 2\theta_i + \theta_{i+1} \end{pmatrix} = \begin{pmatrix} 1 \\ p_r h^2 \\ r \end{pmatrix} \frac{(3R+4)}{3R} \begin{pmatrix} \theta_{i-1} - 2\theta_i + \theta_{i+1} \end{pmatrix}$$

Applying the trapezoidal rule we obtain the Crank-Nicolson method

$$\frac{1}{6k} \left[\frac{(\theta_{i-1}^{n+1} + \theta_i^n)}{2} - 2 \frac{(\theta_i^{n+1} + \theta_i^n)}{2} + \frac{(\theta_{i+1}^{n+1} + \theta_{i+1}^n)}{2} \right] =$$

$$\frac{1}{p_r h^2} \frac{(3R+4)}{3R} \left[\frac{(\theta_{i-1}^{n+1} + \theta_i^n)}{2} - 2 \frac{(\theta_i^{n+1} + \theta_i^n)}{2} + \frac{(\theta_{i+1}^{n+1} + \theta_{i+1}^n)}{2} \right]$$

$$\left[(\theta_{i-1}^{n+1} - \theta_{i-1}^n) + 4(\theta_i^{n+1} - \theta_i^n) + (\theta_{i+1}^{n+1} - \theta_{i+1}^n) \right] =$$

$$\frac{3r}{p_r} \frac{(3R+4)}{3R} \left[\frac{(\theta_{i-1}^{n+1} + \theta_i^n)}{2} - 2 \frac{(\theta_i^{n+1} + \theta_i^n)}{2} + \frac{(\theta_{i+1}^{n+1} + \theta_{i+1}^n)}{2} \right]$$

Simplifying above equation we get

$$\left(1 - \frac{3r}{p_r} \frac{(3R+4)}{3R} \right) \theta_{i-1}^{n+1} + \left(4 + \frac{6r}{p_r} \frac{(3R+4)}{3R} \right) \theta_i^{n+1} + \left(1 - \frac{3r}{p_r} \frac{(3R+4)}{3R} \right) \theta_{i+1}^{n+1} =$$

$$\left(1 + \frac{3r}{p_r} \frac{(3R+4)}{3R} \right) \theta_{i-1}^n + \left(4 - \frac{6r}{p_r} \frac{(3R+4)}{3R} \right) \theta_i^n + \left(1 + \frac{3r}{p_r} \frac{(3R+4)}{3R} \right) \theta_{i+1}^n$$

$h=0.05, k=0.0025$ and $r=1$ the nodal points (y_n, t_n) are shown in the figure (1)

Where $i=1(1)9$ and $n=1, 2, 3, \dots$

The system of equations are solved by using Gauss –Seidel method

Now, we consider equation (8)

$$\frac{\partial u}{\partial t} = G_r \theta + \frac{\partial^2 u}{\partial y^2} - Mu$$



$$\Rightarrow \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G \theta - Mu$$

Applying finite element technique as explained above we obtain

$$(1-3r + \frac{M r h^2}{2}) U^{n+1}_{i-1} + (4+6r+2rM h^2) U^{n+1}_i + (1-3r + \frac{M r h^2}{2}) U^{n+1}_{i+1} =$$

$$(1+3r - \frac{M r h^2}{2}) U^n_{i-1} + (4-6r-2rM h^2) U^n_i + (1+3r - \frac{M r h^2}{2}) U^n_{i+1} + 6 G \theta^n_{r i}$$

The systems of equations are solved by using Gauss –Seidel method.

IV. RESULTS AND DISCUSSIONS

The numerical values of the velocity, skin-friction are computed for different parameters like magnetic field parameter, radiation parameter and time. The purpose of the calculations given here is to assess the effect of the parameter M and R upon the nature of the flow and transport.

The velocity profile for different values of the magnetic field parameters are shown in Fig. 2. It is observed that the velocity decreases in the presence of magnetic field than its absence. This shows that the increase in the magnetic field parameter leads to fall in the velocity. This agrees with expectations, since the magnetic field exerts a retarding force on the free convective flow.

In Fig. 3 the velocity profile are shown for different values of the radiation parameter. It is observed that the velocity increases with decreasing radiation parameter. This shows that velocity decreases in the presence of high thermal radiation.

The velocity profiles for different G_r are shown in the Fig.4 it is observed that velocity increases with increasing value of G_r . As thermal Grashof number increases, the buoyancy effect becomes more significant, as expected; it implies that, more fluid is entrained from the free stream due to the strong buoyancy as G_r increases.

In Fig. 5 temperature profile for different values of R is shown. It is observed that temperature decreases in the presence of radiation.

U 0.5 2 M=2 M=5 M=10 M=15 M=20

Fig. 2—Velocity profiles for different M.

U 0.5 2 R=3 R=5 R=10 R=15 R=20

Fig.3—Velocity profiles for different R

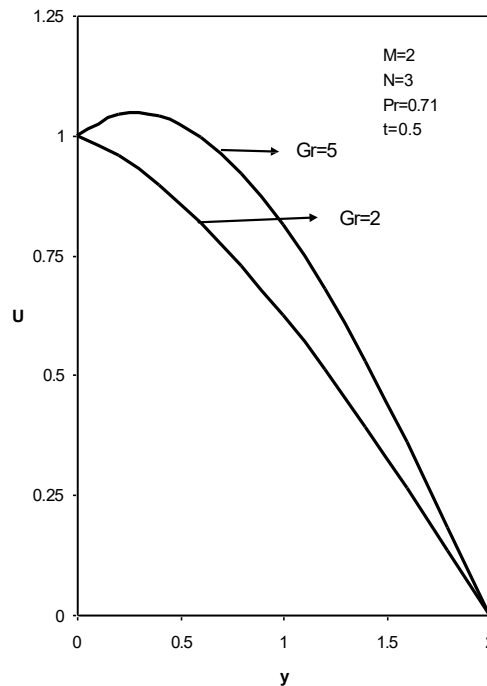


Fig. 4—Velocity profiles for different G_r

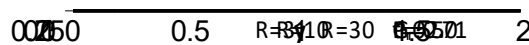


Fig.5—Temperature Profiles for different R

From the velocity field, we study the skin-friction. It is given by

$$\tau = \left. \left(\frac{\partial u}{\partial y} \right) \right|_{y=0}$$

T

The numerical values of τ are presented in table 1. It is observed from the table,

That skin-friction increases with increasing values of the magnetic field parameter. This shows that the wall shear stress increases with increasing magnetic field parameter. This trend is just reversed with respect to time. It is also observed that the skin-friction decreases with increasing Grashof number. It is also observed that the skin-friction increases with increase of Radiation parameter.

Table: 1 –Values of the non-dimensional skin-friction

M	R	$G_r=2$ t=0.5	$G_r=2$ t=0.6	$G_r=5$ t=0.5	$G_r=5$ t=0.6
2	3	1.571	1.4061	1.1558	0.9408
	5	1.5802	1.4163	1.179	0.9664
	10	1.5882	1.4251	1.1989	0.9885
	15	1.5911	1.4283	1.2062	0.9965



5	3	1.5864	1.4206	1.1735	0.9578
	5	1.5956	1.4308	1.1735	0.9833
	10	1.6035	1.4395	1.2162	1.0051
	15	1.6064	1.4427	1.2234	1.0131
10	3	1.6118	1.4446	1.2026	0.9859
	5	1.6209	1.4546	1.2253	1.011
	10	1.6287	1.4633	1.2448	1.0326
	15	1.6315	1.4664	1.2519	1.0404

V. CONCLUSIONS

An exact analysis is performed to study the radiation effects on flow past an impulsively started infinite isothermal vertical plate in the presence of transverse applied magnetic field. The dimensionless governing equations are solved by the Finite Element Method. The effect of magnetic field parameter and radiation parameter are studied. The conclusions are as follows.

- The velocity decreases with the increase of magnetic field.
- The velocity increases with increasing values of G_r .
- The presence of radiation causes a fall in the velocity and temperature.
- Skin-friction increases with increasing values of the magnetic field parameter.
- Skin-friction decreases as the time increases.
- Skin-friction increases with increase of Radiation parameter.

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