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Computational Mathematics and Numerical Techniques

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Abstract- In this paper, we discuss some novel computational methods(bisection method, newton Raphson method, regular false method) in the form of iteration schemes for computing the roots of non-linear scalar equations in a new way. The construction of these iteration schemes is purely basedon The Intermediate Value Theorem,Taylor series expansion, Secant line approximations and interval bracketing. The convergence criterion of the suggested schemes is also given and certified that the newly developed iteration schemes possess quartic convergence order. To analyze the suggested schemes numerically, several test examples have been given and then solved.

Keywords- Bisection method, Regula False Method, algorithms, approximate, analytical solutions, iterative methods, intermediate theorem, interval halving method, root-finding method

I. INTRODUCTION

In computational mathematics, the Bisection Method, Regula False Method, and Newton-Raphson Method are fundamental numerical techniques used to find the real roots of a non-linear equation (f(x) = 0). These iterative algorithms approximate the exact root value within a desired error tolerance when exact analytical solutions are impossible or difficult to obtain.

II. BISECTION METHOD

The Bisection method is one of the simplest and most reliable of iterative methods for the solutions of nonlinear equations.

Definition

The bisection method is used to find the roots of a polynomial equation. It separates the interval and subdivides the interval in which the root of the equation lies. The principle behind this method is the intermediate theorem for continuous functions. It works bynarrowing the gap between the positive and negative intervals until it closes in on the correct answer. This method narrows the gap by taking the average of the positive and negative intervals. It is a simple method and it is relatively slow. The bisection method is also known as interval halving method, root-finding method,

binary search method or dichotomy method.
Let us consider a continuous function "f" which is defined on the closed interval [a, b], is given with f(a) and f(b) of different signs. Then by intermediate theorem, there exists a point x belong to (a, b) for which f(x) = 0.

Bisection Method Algorithm

Follow the below procedure to get the solution for the continuous function:

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For any continuous function f(x),

- Find two points, say a and b such that a < b and f(a)* f(b) < 0
- Find the midpoint of a and b, say "t"
- t is the root of the given function if f(t) = 0; else follow the next step
- Divide the interval [a, b] – If f(t)*f(a) <0, there exist a root between t and a – else if f(t) *f (b) < 0, there exist a root between t and b
- Repeat above three steps until f(t) = 0.

The bisection method is an approximation method to find the roots of the given equation by repeatedly dividing the interval. This method will divide the interval until the resulting interval is found, which is extremely small.

Bisection Method

EXAMPLE : Use the bisection method to find solution accurate to within 10^-2 for the function x^4-2x^3-4x^2+4x+4=0 on [2, 3]]

Solution

Given function is x^4-2x^3-4x^2+4x+4=0

Let f(x) = x^4-2x^3-4x^2+4x+4=0; [2,3]

Now, find the value of f(x) at a_1= 2 and b_1=3

f(2) =2^4-2$(2)^3$-4$(2)^2$+4(2)+4 = 16 - 16 - 16 + 8 + 4 = -16+12= - 4< 0

(f(3) = 3^4-2$(3)^3$-4$(3)^2$+ 4(3) + 4 = 81 - 54 - 36 + 12 + 4 = 7> 0

Since f(2) < 0 and (f(3) > 0

First Iteration

Midpoint of [2, 3] is

$p_1 = \frac{2+3}{2}$ = 2.5

f (p_1) = (f(2.5) =$(2.5)^4$-2$(2.5)^3$-4$(2.5)^2$+4(2.5)+4

f (p_1) = -3.1875 < 0

f (p_1) < 0 and f (a_1) < 0 roots not lies in the interval (p_1, a_1)

f (p_1) < 0 and f (b_1) > 0 0 roots lies in the interval (p_1, b_1)

i.ea_2 = p_1= 2.5 and b_2 = b_1=3

Second Iteration

Midpoint of [2.5, 3] is

$p_2 = \frac{2.5+3}{2}$ = 2.75

f (p_2) = (f(2.75) =$(2.75)^4$-2$(2.75)^3$-4$(2.75)^2$+4(2.75)+4

f (p_2) = 0.3477> 0

f (p_2) > 0 and f (a_2) < 0 roots lies in the interval (a_2, p_2)

a_3 = a_2= 2.5 and b_3 = p_2=2.75

Third Iteration

Midpoint of [2.5, 2.75] is

$p_3 = \frac{2.5+2.75}{2}$ = 2.625

f (p_3) = (f(2.625) =$(2.625)^4$-2$(2.625)^3$-4$(2.625)^2$+4(2.625)+4

f (p_3) = - 1.7576< 0

f (p_3) > 0 and f (a_3) < 0 roots lies in the interval (p_3, b_3)

a_4 =p_3= 2.625 and b_4 =b_3=2.75

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Fourth Iteration

Midpoint of [2.625, 2.75] is

$p_4 = \frac{2.625+2.75}{2} = 2.6875$

f (p_4) = f(2.6875) =$(2.6875)^4$-2$(2.6875)^3$-4$(2.6875)^2$+4(2.6875)+4

f (p_3) = - 0.7956< 0

f (p_4) < 0 and f (a_4) < 0 roots not lies in the interval (p_4, a_4)

f (p_4) < 0 and f (b_4) > 0 roots lies in the interval (p_4, b_4)

i.ea_5 = p_4= 2.6875 and $b_5 = b_4$=2.75

Fifth Iteration

Midpoint of [2.6875, 2.75] is

$p_5 = \frac{2.6875+2.75}{2} = 2.7188$

f (p_5) = (f(2.7188) =$(2.7188)^4$-2$(2.7188)^3$-4$(2.7188)^2$+4(2.7188)+4

f (p_5) = - 0.2466< 0

f (p_5) < 0 and f (a_5) < 0 roots not lies in the interval (p_5, a_5)

f (p_4) < 0 and f (b_4) > 0 roots lies in the interval (p_4, b_4)

i.ea_6 = p_5= 2.6875 and $b_6 = b_5$=2.75

Sixth Iteration

Midpoint of [2.7188, 2.75] is

$p_6 = \frac{2.7188,+2.75}{2} = 2.7344$

f (p_6) = f(2.7344) =(2.7344))^4-2(2.7344))^3-4(2.7344))^2+4(2.7344))+4

f (p_6) = 0.0466> 0

f (p_6) > 0 and f (a_6) < 0 roots lies in the interval (a_6, p_6)

i.ea_7=a_6= 2.7188 and $b_7 = p_6$=2.7344

Seventh Iteration

Midpoint of [2.7188, 2.7344] is

$p_7 = \frac{2.6875+2.7344}{2} = 2.7266$

f (p_7) = f(2.7266) = (2.7266))^4-2(2.7266))^3-4(2.7266))^2+4(2.7266))+4

f (p_7) = - 0.1025< 0

f (p_7) < 0 and f (a_7) < 0 roots not lies in the interval (p_7, a_7)

f(p_7) < 0 and f (b_7) >0 roots lies in the interval (p_7, b_7)

i.ea_8= p_7= 2.7266 and $b_8 = b_7$=2.7344

$p_8 = \frac{2.7266+2.7344}{2} = 2.7305$

Here p_6= p_8= = 2.73(correct within 10^{-2}

Therefore the root of the equation is 2.73

III. BISECTION METHOD CONVEGENCE

The Bisection Method is guaranteed to converge to a root of a continuous function, provided the initial interval contains a sign change. The method possesses a linear convergence rate of 0.5, meaning the maximum possible error is halved at every single iteration.

Advantages

- Always converges if sign change exists
- Very easy to calculate and program

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- Error can be estimate accurartely

Disadvantages
- The convergence of the bisection method is slow as it is based on halving the interval.
- If one of the initial guesses is closer to the root, it will take a larger number of iterations to reach the root.
- If a function is such that it just touches the x-axis

IV. REGULAR FALSE METHOD OR FALSE POSITION METHOD

This method is an iterative root-finding algorithm that combines features of the bisection method and the secant method to approximate the real roots of a non-linear equation (f(x) = 0).

Formula
The method requires two initial guesses, $x_0 and \ x_1$ .such that the function changes signs over the interval, meaning f (x_0). f (x_1) < 0. The next approximation x_2 is found using the formula:
$x_2 = x_1 - \frac{f(x_1).(x_1 - x_0)}{f(x_1) - f(x_0).}$
Alternatively, it can be written as:

Algorithm:
- Find two points)and$x_0 and \ x_1$such that f (x_0). f (x_1) < 0).
- Calculate Next Root: Compute$x_2$using the Regula Falsi formula.
- Check Tolerance: If |f(x_2)<0 stop $x_2$is the root.
- Update Interval:
- If (f(x_0). f(x_2)< 0, the root lies in ([x_0, x_2]. Set ($x_1 = x_2$).
- If (f(x_0). f(x_2) > 0), the root lies in ([x_2, x_1]). Set ($x_0 = x_2$).
- **Repeat**: Return to Step 2 until convergence is reached.

Example:
Find the real root of the equation (f(x) = x^3 - x - 1 = 0 using the False Position Method.

Establish Initial Boundaries
Evaluate integers to locate a sign change:
f(1) = 1^3 - 1 - 1 = -1 < 0
(f(2) = 2^3 - 2 - 1 = 5> 0
Since (f(1). f(2) < 0
, the real root lies strictly inside the interval [1, 2]
Set x_0 = 1 and x_1 = 2
First Iteration:
(x_0 = 1), f(x_0) = -1
(x_1= 2), f(x_1) = 5
$x_2 = x_1 - \frac{f(x_1).(x_1 - x_0)}{f(x_1) - f(x_0).}$
$x_2 = 2 - \frac{5(2-1)}{5-(-1)} = 2 - \frac{5}{6}$
$= \frac{12-5}{6} = \frac{7}{6} \approx 1.16667$
$x_2 \approx 1.16667$

Evaluate *f*(x_2)
f(1.16667)≈ (1,16667)^3 – 1.16667 - 1 ≈ -0.57870< 0

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Second Iteration:

Because f(1.16667)\) is negative, it replaces the old negative boundary x_0

The updated bracket interval is [1.16667, 2]

$x_3 = x_2 - \frac{f(x_2).(x_2 - x_1)}{f(x_2) - f(x_1).}$

$x_3 = 1.16667 - \frac{-0.57870.(1.16667-2)}{-0.57870-5}$

$= 1.16667 - \frac{-0.57870.(1.16667-2)}{-0.57870-5}$

$= 1.16667 - \frac{(-0.57870.).(-0.83333)}{-5.57870}$

$= 1.16667 - \frac{0.48224807}{-5.57870}$

= 1.16667 – (-0,0864445)

= 1.25311

Evaluate the function at this point

f(1.25311) = $(1,25311)^3$ – 1.25311 - 1

≈-0.28536

Final Answer

The real root of the equation $x^3 - x - 1 = 0$ is approximately 1.25311

Conclusion

The root of the equation $x^3 - x - 1 = 0$ after two iterations of the RegulaFalsi method is approximately **1.25311**

Advantages

- This method is very easy because it is very basic in its structure.
- This bracketing method can guarantee the convergence if the function is continuous within the interval.
- Unlike the Newton-Raphson method, the RegulaFalsi method does not require the computation of derivatives.

Disadvantages

- RegulaFalsi, compared to other methods like Newton-Raphson may be slow to converge.
- The speed of convergence depends heavily on the choice of initial interval.
- The method can sometimes exhibit oscillatory behaviour around the root, leading to slow convergence.

V. CONCLUSION

The Bisection method and RegulaFalsi (False Position) method are both reliable bracketing techniques used to find real roots of continuous non-linear equations. While both methods require an initial interval where the function changes signs ((f(a) .f(b))< 0 they differ significantly in their approach, efficiency, and predictability.

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