

A Comprehensive Review on Role of Linear Algebra in Artificial Intelligence and various Engineering Applications

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Abstract- AI is growing much faster in every field including engineering and linear algebra has provided a strong base for it. Linear Algebra is a core branch of mathematics which is used in mostly all engineering fields. This review paper carries out its applications on five major fields of engineering, i.e., civil engineering, electrical engineering, mechanical engineering, computer science engineering, and chemical engineering. Vectors, matrices, system of linear equations, linear transformation, and eigenvalues have been discussed here. Linear algebra can be seen in civil engineering through structural analysis with the help of finite element methods and hydraulic modeling. In electrical engineering, it has a big part in circuit analysis, signal processing, control systems as well as communication; while in mechanical engineering it is applied for vibration studies robotics and fluid dynamics. In chemical engineering modeling processes thermodynamics and transport phenomena take place with the help of linear algebra. In computer science linear algebra helps machine learning data analysis computer graphics and cryptography to advance further. This paper illustrates the critical role that linear algebra plays as an underlying structure supporting AI and ML systems by their use of vector and matrix-based data representation as well as model computation. It reveals how different techniques such as neural networks dimensionality reduction optimization are based on fundamental principles from linear algebra to handle large datasets in modern engineering fields. Hence Linear algebra is important for new ideas and tech progress in engineering.

Keywords: Linear Algebra, Engineering Applications, Matrices and Vectors, Eigenvalues, Machine Learning.

I. INTRODUCTION

AI is developing much faster in recent times but the foundation of it was only possible because of linear algebra. Linear algebra is one of the tools or fundamental concepts most widely used in engineering. It has core concepts like vectors, vector spaces, matrices, systems of linear equations, linear transformations and eigenvalues and eigenvectors [1]. Some advanced concepts like canonical forms (used to simplify complex matrices in simple form to understand them easily), tensors (used to generalize the concepts of scalar and vector), and infinite dimensional spaces are also obtained from linear algebra [2]. These concepts also help engineers to present practical systems in a structured mathematical form and help in the application of mathematical operations to get accurate and efficient solutions. Also, linear algebra got advancements in numerical methods for solving

large systems and computational tools (MATLAB, NumPy, and SciPy) [3]. Linear algebra plays the most important role in areas like large scale computations and simulations using the core concepts like matrix operations, eigenvalues, and numerical linear algebra as talked by Larson et al.[4].

As I shared in the starting that AI is growing much faster, and the foundations of AI is also built by linear algebra as it helps AI to do calculations, running simulations, and managing data. Linear algebra in AI increases the calculation speed by simplifying the complex calculations, making the model much faster in image generation, analysing simulations and using them to predict next stage of any scientific model faster, and also training model has become much faster now a days by consistent improvements and innovations [5]. In addition, Artificial Intelligence and Machine Learning models are based on linear algebraic structures. The data is represented in

vector spaces and computations are performed using matrix operations.

Neural networks, principal component analysis, and regression model algorithms use eigenvalues, matrix factorization, and linear transformations extensively. This deep integration shows critical role of linear algebra playing in the development of intelligent systems for data-driven decision making. The application of linear algebra can spread by the continuous growth of technology and innovations. Hence, linear algebra has become a necessity for various branches of engineering and in other fields [6]. This review aims to provide a detailed study of linear algebra applications in different engineering fields with its advancements. This study will show how linear algebra connects different fields of engineering. It also discusses role of linear algebra in different AI applications and concepts.

II. LITERATURE REVIEW

Overview of Role of Linear Algebra in Various Engineering Fields:

Strang, G.[7] explained that linear algebra is the main and most important framework for solving engineering problems. It includes main concepts like vectors, vector spaces, matrices and their operations, systems of linear equations, linear transformations, eigenvalues, and eigenvectors. Some of these concepts are: Vectors: Vector is a quantity with size and direction. The formulas for a vector $\vec{A} = a_i\hat{i} + b_j\hat{j} + c_k\hat{k}$ is given as Equation (1)[7] and unit vector as shown in Equation (2).

$$|\vec{A}| = \sqrt{a^2 + b^2 + c^2} \quad (1)$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} \quad (2)$$

Vector Spaces is a group of vectors that satisfy some properties. It is closed under addition and scalar multiplication. Matrix is a rectangular array of numbers or other mathematical objects with elements or entries arranged in rows and columns, usually satisfying certain properties. Matrix operations are operations which are applied on matrices and used to manage data, solve linear equations and transformations are called matrix operations. System of linear equations refers to a set of two or more linear equations having the same variables. Linear transformation is a type of function

$T:V \rightarrow W$ that maintains the vector addition and scalar multiplication operations between two vector spaces, V and W . Its formula is given as Equation (3) and (4) [7].

$$T(u + v) = T(u) + T(v), \quad (3)$$

$$T(\lambda u) = \lambda T(u) \quad (4)$$

Eigenvalues and Eigenvectors are among the basic principles in linear algebra which help to understand the effect of linear transformations on certain vectors. It is given as Equation (5), where A is the matrix, v is the eigenvector, and λ is the eigenvalue. These concepts give the foundation for various applications of linear algebra in engineering. These help engineers to model and analyse complex systems and in solving complex problems. In civil engineering, linear algebra is very important for structural analysis.

$$Av = \lambda v \quad (5)$$

Przemieniecki, J. S.[8] told that the most important equation for structural analysis is Equation (6) where K is the global stiffness matrix, u is the displacement vector, and F is the force vector. To form the global stiffness matrix using the element stiffness matrices, the process requires summing up the matrices and transforming the coordinates.

$$K \cdot u = F \quad (6)$$

Bathe, K. J.[9] said that popular methods for this include finite element procedures, Cholesky factorization, conjugate gradient methods, and multigrid algorithms. Chopra, A. K.[10] has shared that to do modal analysis on structures, then need to solve the generalized eigenvalue problem as given in equation (7), in this case, M stands for the mass matrix, ϕ stands for mode shapes (eigenvectors), and ω stands for natural frequencies (which are the same as eigenvalues). This analysis is very important for earthquake engineering, wind engineering, and vibration serviceability assessment.

$$K\phi = \omega^2 M\phi \quad (7)$$

Ghilani, C. D.[11] has shared that least squares adjustment is a method used in surveying to find the best coordinates from measurements that are too close together. This is achieved by resolving systems of linear equations characterized by a surplus of

equations relative to unknowns. The normal equation method is used, which looks like equation (8) [11] where W is a weight matrix that shows how accurate the measurements are. Rossman, L. A.[12] explained that, for water supply networks, they solve linear equations that come from mass balance at each point and energy balance in the pipes. People often use the Hardy Cross method or Newton Raphson method for this.

$$A^T W A \cdot x = A^T N \cdot b \quad (8)$$

Hayt et al.[13] described how linear algebra is the main math tool in electrical engineering. It helps to study, model, and simulate electrical and electronic systems. It is used in circuits, signal processing, control systems, communication, and electromagnetics. Kirchhoff's laws turn circuit problems into systems of linear equations in matrix form. Nodal analysis gives $G \cdot v = i$, where G is the conductance matrix, v is the node voltage vector and i is the current source vector. This gives an $(n-1) \times (n-1)$ [13] system for the n node circuits. Nagel, L. W.[14] in technical report mentioned Modified Nodal Analysis (MNA) handles voltage sources too and is used in SPICE(Simulation Program with Integrated Circuit Emphasis) software.

Oppenheim et al.[15] described that Digital signal processing uses linear algebra a lot. Some main uses are Discrete Fourier Transform (DFT) which is a matrix times a vector using a unique DFT matrix. It is shown as $X = W \cdot x$ [15], where W is the DFT matrix with entries. Haykin, S.[16] in Adaptive Filter Theory resolves the equation (9) [16]. Digital filters (FIR and IIR) use Toeplitz matrices and matrix multiplication for convolution. Adaptive filters: The Wiener filter and LMS(Lambda Mu Sigma) method solve an equation (10) [16] where R using the autocorrelation matrix is a cross-correlation vector.

$$W_{kn} = e^{-j2\pi kn/N} \quad (9)$$

$$R \cdot w = p \quad (10)$$

Ogata, K.[17] presented state space models for linear systems use matrix equations (11) [17] State space models for linear systems use matrix equations. They check if the system can be controlled or observed by looking at matrix ranks. Stability comes from the eigenvalues of matrix A [B AB A² B...A⁽ⁿ⁻¹⁾ B] [17].

Sadiku, M. N. O.[18] explained when they turn Maxwell's equations into numbers with finite difference or finite element, they get big systems of linear equations. The Method of Moments gives dense matrices for antennas, while FDTD gives sparse ones. Eigenvalue analysis finds resonant modes in cavities and waveguides. Tse et al.[19] talked about MIMO (Multiple Input Multiple Output) systems are shown as $y = Hx + n$, where H is the channel matrix. They use singular value decomposition (SVD) of H to find channel capacity and best ways to send signals using eigenvalues.

$$\dot{x} = Ax + Bu, y = Cx + Du \quad (11)$$

Linear algebra is a basic tool in mechanical engineering. It helps to study moving systems, structures, robots, and fluids by turning them into simple matrix equations that computers can solve fast. Meirovitch, L.[20] explained that the motions equation for a system with many degrees of freedom is as given by equation (12) [20] When there is no force or damping($F=0, C=0$), it becomes a generalised eigenvalue problem that gives natural frequencies and mode shapes. For forced vibration, they use modal superposition based on eigenvectors. Reddy, J. N.[21] Similarly like in civil engineering, they solve big linear systems for stress, heat, and combined problems. In mechanical work they also deal with nonlinear materials (using Newton Raphson), contact, and coupled thermal structural cases.

$$M\ddot{x} + C\dot{x} + kx = F(t) \quad (12)$$

Craig, J. J.[22] discussed that Robot kinematics extensively uses transformation matrices. The homogeneous transformation matrix combining rotation and translation is shown in equation (13) [22]. Forward kinematics involves multiplying successive transformation matrices (Denavit Hartenberg convention), while inverse kinematics may involve solving systems of nonlinear equations through linearization. Siciliano et al.[23] The Jacobian matrix J relates joint velocities to end effector velocities as given in equation (14) [23]. Singularity analysis involves examining the rank and determinant of J .

$$T = \begin{bmatrix} R & d \\ 0^T & 1 \end{bmatrix} \quad (13)$$

$$\dot{x} = J(q) \cdot \dot{q} \quad (14)$$

Hughes et al.[24] explained that linear algebra is the backbone of modern computer engineering and computer science. It helps to represent, study, and improve complex computer systems in machine learning, data work, search, graphics, vision, and cryptography. 3D graphics uses 4×4 matrices for moving, rotating, scaling, projecting, and viewing. The computer does millions of matrix vector multiplications every frame, which is why GPUs are made for fast linear algebra. Goodfellow et al.[25] studied that linear algebra is arguably the mathematical backbone of modern machine learning: Neural Networks: Forward propagation through a layer is $a = \sigma(Wxb)$ [25], where W is the weight matrix. Training via backpropagation involves computing matrix gradients.

Felder et al.[26] connected the chemical engineering with linear algebra and explained that linear algebra helps chemical engineers to model, study, and improve chemical processes. It is used in balances, reactor design, thermodynamics, transport, and control. Steady state material balance equations for multi component, multi-unit chemical processes form systems of linear equations: $A \cdot x = b$ [26] where A is the coefficient matrix derived from stoichiometric and flow relationships, x is the vector of unknown flow rates or compositions, and b represents known inputs. The rank of A determines whether the system is fully specified, underspecified, or overspecified.

Aris, R.[27] gave stoichiometric analysis of complex reaction networks uses matrix methods. The stoichiometric matrix S , where entry S_{ij} represents the stoichiometric coefficient of species i in reaction j , enables systematic analysis of reaction independence and species balances $dc/dt = S \cdot r$ [27] where c is the concentration vector and r is the reaction rate vector. The null space of S determines conservation relationships. Bristol, E. H.[28] explained that how process control uses state space methods. The Relative Gain Array for pairing inputs and outputs is given in equation (15) [28], where $\circ$ denotes element wise multiplication. Model Predictive Control solves linear problems in real time. Smith et al.[29] introduced equation of state calculations and phase equilibrium computations often involve solving systems of nonlinear equations

through Newton Raphson iteration, which requires assembling and solving Jacobian based linear systems at each iteration given in equation (16) [29].

$$\Delta = K \circ (K^{-1})^T \quad (15)$$

$$J(x_k) \cdot \Delta x = -f(x_k) \quad (16)$$

Heat, mass, and flow equations in chemical equipment give large sparse linear systems, same as in mechanical CFD. Bird et al.[30] enlightened about discretization of partial differential equations governing heat transfer, mass transfer, and fluid flow in chemical engineering systems (reactors, heat exchangers, separation columns) produces large sparse linear systems similar to those in mechanical engineering CFD applications.

Study of Applications of Linear Algebra in Artificial Intelligence:

According to LeCun et al [31], modern Artificial Intelligence systems, especially deep layers, fundamentally rely on linear operations. Neural networks take input data in the form of vectors and matrices and each layer uses the two live transformations of matrix multiplication and bias addition as shown in Fig1. Linear algebra computations optimize efficient and scalable computations of heterogeneous architecture for AI model computations. Thus, linear algebra is one of the important mathematics for AI. Also, deep learning was seen to achieve effective results with images and sound by using CNNs.

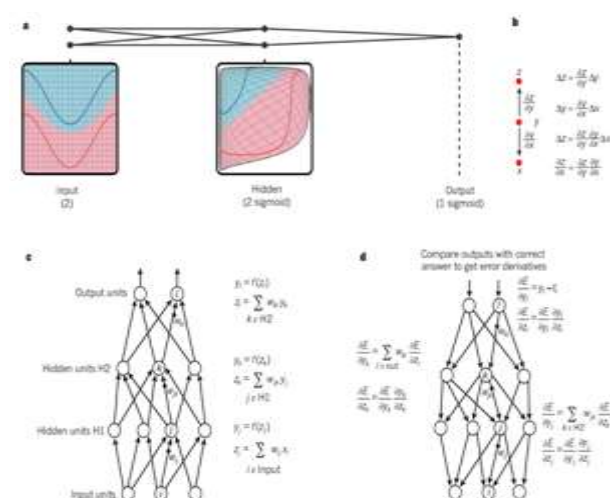


Fig.1. Linear Algebra used in Multilayered Neural network and back propagation [31]

The research of Murphy, K. P. [32] showed that probabilistic machine learning models use linear algebra to process high dimensional data. The algorithms which include Gaussian Mixture Models and Bayesian networks use covariance matrices and vector spaces as well as matrix decompositions. AI systems use linear algebra to process and depict uncertainty in their operations. The work established a complete system which combines probability theory with linear algebra to create strong AI systems. They explained the matrix-based algorithms which are used for inference and learning because they provide fast execution. The book included real-world case studies which showed how probabilistic models function in pattern recognition and decision-making systems.

Shalev-Shwartz, S. and Ben-David, S. [33] showed that supervised learning algorithms including linear regression and logistic regression and support vector machines use linear algebra as their core mathematical foundation. The models use vector data representations together with matrix optimization methods to decrease loss and enhance prediction results. Their research examined machine learning theoretical principles which included generalization and overfitting and risk reduction methods. The researchers proved that learning algorithms' optimization challenges could be addressed through linear algebra methods. The book contained scientific proofs and algorithmic frameworks which demonstrated how learning efficiency depends on matrix usage.

According to Hastie, Tibshirani, and Friedman [34], statistical learning methods are based on linear algebra for optimization problem solving. Methods like least squares, ridge regression, and PCA make clever use of Eigen values/matrix factorizations to reduce dimensionality and sharpen the focus on data. Their study presented various popular statistical models and gave insight into their computational foundations using linear algebra. The role of eigen decomposition and singular value decomposition in reducing features and noise was also demonstrated using real world case studies to improve performance.

Deep learning models require advanced matrix computation for its forward and backward propagation [35]. Neural networks store their weights and biases as matrices, and those matrices are updated through training in a linear fashion with regards to gradient calculations using linear algebra. They systematically explained the architecture of deep neural networks and the math behind training them. It explained how to compute gradients using matrix derivatives and that optimization algorithms (like gradient descent) depend on vectorized operations. The challenges posed by vanishing gradients and computational complexity are studied in the book.

Linear algebra is vital in pattern recognition and machine learning technologies for features extraction and transformation as observed C. M. Bishop [36]. Eigen decomposition and singular value decomposition (SVD) can help identify important features in a dataset which help improve the performance of the model. The research work of Bishop presented a probabilistic view of pattern recognition using statistics and linear algebra as shown by Fig2. He showed how dimensionality reduction methods improve classification accuracy. The additional components of the article consist of information clustering, classification, and density estimation.

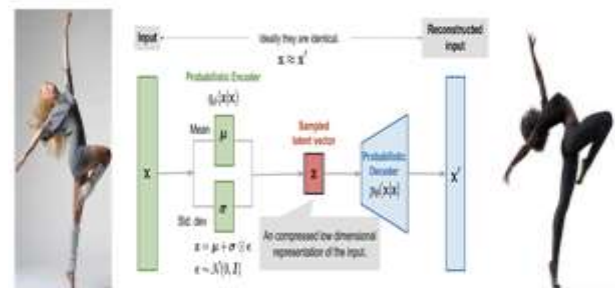


Fig.2. Working of Variational Auto-Encoder using probabilistic code [36]

Deisenroth et al [37] pointed out that reinforcement learning algorithms also rely on linear algebra for value function approximation and policy optimization. Matrix operations are used to update policies and determine expected rewards in dynamic environments. They sought to bridge mathematics and machine learning, explaining the core ideas in an

intuitive manner. They showed how linear algebra can be used in reinforcement learning to solve MDPs.

Linear algebra is useful because it allows efficient computations as supported by G. Strang [38] in the context of AI implementations and architecture. Understanding how data travels through machine learning models requires that one should know concepts like vector spaces and linear transformations. Strang's contributions linked classical linear algebra with data science applications. He discussed that matrix factorizations and eigenvalues are useful for data analysis and machine learning.

Zhang et al [39] said that modern AI frameworks and libraries optimize their functions for matrix and tensor operations through their design. The frameworks use linear algebra to improve training efficiency through GPU and distributed computing, which enables developers to create large scale AI systems. The research showed how to build deep learning systems by using linear algebra techniques for efficient model development. The system demonstrated neural network functionality together with optimization methods and data handling workflows. The research demonstrated that parallel matrix computations could enhance both scalability and performance of their system.

Linear algebra serves as the fundamental base which supports convex optimization that enables most machine learning algorithms to function as studied by Boyd et al [40]. Matrix based techniques deliver faster convergence times and greater stability during model training and are thus the primary method for solving AI optimization problems. This work established a comprehensive mathematical framework which supported the optimization methods applied in artificial intelligence. They used linear algebra methods to demonstrate how to solve convex optimization problems with high efficiency.

III. CONCLUSION

This study shows that linear algebra is needed in every big engineering field – civil, electrical,

mechanical, computer science, and chemical. The simple ideas of matrices, vectors, eigenvalues, and linear equation systems are used to model, study, and solve real engineering work. Different types of advancements help in the analysis of data to solve actual problems.

The review explains how linear algebra is connected to practical things like structural checks, circuit design, vibration study, machine learning, fluid flow, and process control. It turns big complicated systems into easy math forms that computers can handle. With the growth of technology, problems also become harder. Hence, linear algebra is becoming even more important, mostly in AI, data science, and advanced simulations. Moreover, with the fast growth of Artificial Intelligence and Machine Learning, linear algebra has become more important.

It is the computational foundation of modern intelligent systems. Artificial intelligence uses concepts like matrices and vectors to make predictions. This close link ensures that linear algebra will play a significant role in future tech advancements. Linear algebra is not just a theory but actually a useful tool. Engineers use it to understand problems and find solutions. It helps engineers work efficiently and come up with new ideas. Linear algebra is necessary for people who study and, for people who work in industries.

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