

MHD Free Convection Laminar Flow of a Viscous Fluid Through a Vertical Porous Plate

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Abstract- This study investigates the steady, two-dimensional MHD (magnetohydrodynamic) free convection flow of an incompressible, viscous, and electrically conducting fluid past a vertical porous plate. The flow is subjected to a uniform transverse magnetic field acting perpendicular to the direction of flow. This study investigates the steady, two-dimensional MHD (magnetohydrodynamic) free convection flow of an incompressible, viscous, and electrically conducting fluid past a vertical porous plate. The flow is subjected to a uniform transverse magnetic field acting perpendicular to the direction of flow. The problem is governed by a coupled non-linear system of partial differential equation. In this case exact solutions are not possible, by the explicit finite difference method. The governing equations—consisting of continuity, momentum, and energy balance—are transformed into a system of non-linear ordinary differential equations using appropriate similarity variables. The velocity distribution, the temperature distribution, coefficient of skin friction and rate of heat transfer has been investigated.

Keywords: Magnetic parameter (M), porosity parameter (k), Magnetic parameter (Hartmann number), Grashof number, Prandtl number, and Permeability of the porous medium, on the velocity and temperature.

I. INTRODUCTION

This chapter analyzed the effect of MHD free convection Couette flow through of a viscous incompressible fluid past an infinite vertical plate through porous with account viscous dissipative heat, of an uniform transverse magnetic field.

Perdikis (1983) studied the flow of a viscous fluid through a porous medium by a vertical surface. H. A. Attia (2002) studied Unsteady MHD flow and heat transfer of dusty fluid between parallel plates with variable physical properties.

H. A. Attia (2006) discussed Unsteady MHD Couette flow and heat transfer of dusty fluid with variable physical properties. Applied Mathematics and Computation.

Singh et al. (1985) study the free convection flow through a porous medium. Raptis (1983) study the unsteady flow through porous medium bounded by an infinite porous plate subject to a constant suction and variable temperature. Kumar (2004) discussed the Hall current effect on MHD free convection flow through porous media past a semi-infinite vertical plate with mass transfer. Muhammad (2005)

discussed the Effects of Hall current and heat transfer on flow due to a pull of eccentric rotating.

Mathematical Formulation

The effect of MHD free convection Couette flow through of a viscous incompressible fluid past an infinite vertical plate through porous with account viscous dissipative heat, of an uniform transverse magnetic field.

The problem is governed by a coupled non-linear system of partial differential equation. So we employ explicit finite difference method for its solution.

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho} \frac{\partial p}{\partial y} + g\beta(T - T_\infty) - \frac{\sigma \mu B^2}{\rho} u - \frac{\nu}{k} u \dots (1)$$

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial t} \right)^2 \dots (2)$$

Where u^* is the velocity of the fluid, k is the thermal conductivity of fluid, ρ is the density of fluid, ν is the kinematics viscosity, β is the coefficient of volume of expansion, T_0 is the temperature of lower plate, T_∞ is the temperature of fluid far away from the plate. μ is the viscosity of fluid, σ is the electrical conductivity of the fluid, μ_0 is the magnetic permeability, C_p is the

specific heat of constant pressure, k - permeability of porous medium.

Boundary conditions are:

$$(3) \left. \begin{aligned} t^* \leq 0, u^* = 0, T^* = T_\infty^* \quad \text{for all } y^* \\ t^* > 0, u^* = 0, T^* = T_w^* \quad \text{at } y^* = 0, \\ u^* \rightarrow 0, T^* = T_\infty^* \quad \text{at } y^* \rightarrow 0, \end{aligned} \right\}$$

The non-dimensional quantities a

$$\begin{aligned} t &= \frac{t^*}{T_R}, \quad y = \frac{y^*}{L}, \quad u = \frac{u^*}{U_0} \quad \text{and} \quad \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \\ \Delta T &= T_w^* - T_\infty^* \\ U_0 &= (\nu g \beta \Delta T)^{\frac{1}{3}} \quad \text{Re ference velocity,} \\ L &= \left(\frac{g \beta \Delta T}{\nu^2} \right)^{-1/3} \quad \text{Re ference length,} \\ T_R &= \frac{(g \beta \Delta T)^{-2/3}}{\nu^{-1/3}} \quad \text{Re ference time,} \\ P_r &= \frac{\mu C_p}{K} \quad \text{prandtl numbers,} \\ E &= \frac{U_0^2}{C_p \Delta T} \quad \text{ec ker t number,} \end{aligned} \quad \dots (4)$$

$$\begin{aligned} M &= \frac{\sigma \mu_e^2 H_0^2 T_R}{\rho} \quad \text{magnetic parameter,} \\ K &= \frac{K^*}{\nu T_R} \quad \text{porosity parameter,} \end{aligned}$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \frac{T_R}{\mu} \frac{\partial p}{\partial y} + \theta - \left(M^2 + \frac{1}{K} \right) u$$

... (5)

And

$$P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + EP_r \left(\frac{\partial u}{\partial y} \right)^2$$

... (6)

The non-dimensional initial boundary conditions is

$$(7) \left. \begin{aligned} t \leq 0, \quad u = 0, \quad \theta = 0, \quad \text{for all } y \\ t < 0, \quad u = 0, \quad \theta = 1, \quad \text{at } y = 0 \end{aligned} \right\}$$

Numerical solution:

Now define a new dimensional variable

$$\eta = \frac{y}{1+y} \Rightarrow y = \frac{\eta}{1-\eta}$$

... (8)

Using the equation (8), the equation (5) and (6) reduced in the form

$$\frac{\partial u}{\partial t} = (1-\eta)^4 \frac{\partial^2 u}{\partial \eta^2} - 4(1-\eta)^3 \frac{\partial u}{\partial \eta} - (1-\eta)^2 \frac{T_R}{\mu} \frac{\partial p}{\partial \eta} + \theta - Mu - \frac{1}{k} u$$

... (9)

And

$$P_r \frac{\partial \theta}{\partial t} = (1-\eta)^4 \frac{\partial^2 \theta}{\partial \eta^2} - 4(1-\eta)^3 \frac{\partial \theta}{\partial \eta} + P_r E (1-\eta)^4 \left(\frac{\partial u}{\partial \eta} \right)^2$$

... (10)

And the initial boundary conditions are

$$(11) \left. \begin{aligned} t \leq 0, \quad u = 0, \quad \theta = 0, \quad \text{for all } \eta \\ t > 0, \quad u = 0, \quad \theta = 1, \quad \text{at } \eta = 0 \\ \text{and } u = 0, \quad \theta = 0, \quad \text{at } \eta = 1 \end{aligned} \right\}$$

... (11)

Equations (9) and (10) are coupled non-linear partial differential equations and are to be solved by using the initial boundary condition (XI).

Using the finite difference Technique the equations (9) and (10) becomes in the form

$$\begin{aligned} \frac{u_{i,j+1} - u_{i,j}}{\Delta t} &= \theta_{i,j} + (1-\eta_{i,j})^4 \left\{ \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\Delta \eta)^2} \right\} - 4(1-\eta_{i,j})^3 \left(\frac{u_{i+1,j} - u_{i,j}}{\Delta \eta} \right) \\ &\quad - \left(M + \frac{1}{k} \right) u_{i,j} - (1-\eta_{i,j})^2 \frac{T_R}{\mu} \left(\frac{p_{i+1,j} - p_{i,j}}{\Delta \eta} \right) \end{aligned}$$

... (12)

$$\begin{aligned} P_r \left(\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} \right) &= (1-\eta_{i,j})^4 \left\{ \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta \eta)^2} \right\} - 4(1-\eta_{i,j})^3 \left(\frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta \eta} \right) \\ &\quad + (1-\eta_{i,j})^4 P_r E \left(\frac{u_{i+1,j} - u_{i,j}}{\Delta \eta} \right)^2 \end{aligned}$$

... (13)

The initial condition (11) we have the following form

$$(14) \left. \begin{aligned} u(0,0) = 0, \quad \theta(0,0) = 1, \\ u(i,0) = 0, \quad \theta(i,0) = 0, \end{aligned} \right\}$$

... (14)

And the boundary condition (11) are expressed by using finite difference form as follows

$$\left. \begin{aligned} \theta(0, j) = 0, \quad \theta(0, j) = 1, \text{ for all } j \\ u(1, j) = 0, \quad \theta(1, j) = 0, \text{ for all } j \end{aligned} \right\}$$

... (15)

We now calculate the skin friction from the velocity field. It is given in the non-dimensional form as

$$\tau = - \left(\frac{du}{d\eta} \right), \quad \text{where } \tau = \frac{\tau^1}{\rho U_0^2}$$

... (16)

Result and Discussion: The velocity profile represent in fig-1 having at the value at $pr = .7$, $E = .25$ and different value of Magnetic parameter (M), porosity parameter (K) and time (t). From fig-1, the velocity of fluid increases first near the plate and then the trend gets reversed as η increases. It is also noticed that velocity increases with the increase in K and t, but it decreases with increase in M. The skin friction profile represent in fig-2 for the same values as taken in case of velocity.

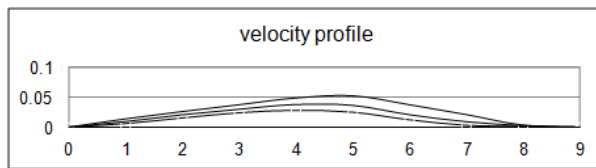


Fig - 1

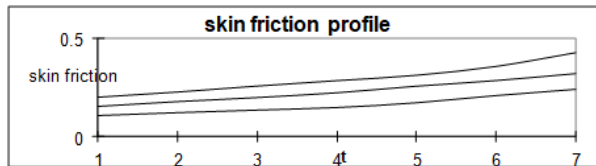


Fig - 2

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