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Mathematical Modeling of Atmospheric Pollutant Dispersion Under Periodic Emissions: Implications for Respiratory and Cardiovascular Health

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Abstract- This study presents a mathematical model for the dispersion of atmospheric pollutants subjected to periodic emission sources and removal dynamics. Using an advection-diffusion-reaction framework, we derive and analyze the governing partial differential equation incorporating a sinusoidal source term and a constant atmospheric removal rate. The model captures real-world conditions such as diurnal emission cycles and steady pollutant decay. Analytical and numerical solutions are explored to understand the spatiotemporal behavior of pollutant concentrations. Numerical results highlight that pollutant concentration profiles evolve with time, showing spatial spreading, downstream advection, and amplitude attenuation due to decay. Furthermore, increasing the emission oscillation amplitude (α) leads to more pronounced temporal fluctuations in concentration at fixed locations. Importantly, given that the World Health Organization reports that air pollution contributes to nearly 7 million premature deaths annually, primarily due to respiratory and cardiovascular diseases, this work underscores the critical need for accurate pollution modeling to inform effective monitoring and mitigation strategies under oscillatory emission conditions.

Keywords - Pollutant dispersion, periodic emission, advection-diffusion-reaction, Green's function, atmospheric modelling, removal dynamics.

I. INTRODUCTION

According to WHO, air pollution contributes to 7 million premature deaths every year, most of them due to respiratory and heart diseases. Pollutant dispersion in the atmosphere is a critical area of study in environmental science, applied mathematics, and public health. Understanding the transport and spread of pollutants such as particulate matter, greenhouse gases, volatile organic compounds, and other airborne contaminants is essential for evaluating environmental quality, mitigating health risks, and designing effective regulatory policies [2,23,30].

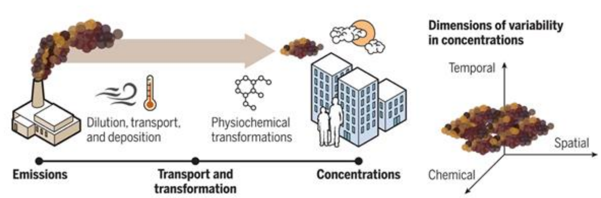

Figure (1): Pathway of Air Pollutant Emissions to Ambient Concentrations

The behavior of pollutants in the atmospheric boundary layer is governed by a complex interplay of factors including emission sources, wind advection, turbulent diffusion, chemical transformations, gravitational settling, and various removal mechanisms such as deposition and precipitation [Figure (1)]. Traditional atmospheric dispersion models, including Gaussian plume models and deterministic mathematical formulations, often rely on simplifying assumptions such as steady-state emissions, uniform wind velocities, or spatial homogeneity [5,18,22,33]. While these assumptions provide analytical tractability, they can fall short in capturing the true variability and dynamics of pollutant sources in real-world scenarios [10,17,21]. For instance, vehicular traffic on urban roads follows diurnal cycles with peak emissions during morning and evening hours. Industrial plants may operate in shifts or have periodic discharges depending on production schedules. Likewise, power generation from renewable sources like biomass combustion or fossil fuel-based plants may also introduce cyclic emission patterns [9,20].

Consequently, assuming a constant emission rate in such contexts can lead to inaccuracies in predicting pollutant concentrations and exposure risks. Another important factor influencing pollutant behavior is the process of removal from the atmosphere. Removal mechanisms can be broadly categorized into physical processes such as gravitational settling, wet and dry deposition, and chemical processes including reactions with atmospheric constituents like ozone or hydroxyl radicals. These removal processes, too, are not necessarily constant over time. For instance, precipitation events can cause episodic yet significant reductions in airborne particle concentrations, while diurnal temperature variations can modulate chemical reaction rates and therefore affect removal efficiency [13,29,34]. Incorporating such temporal variations into pollutant dispersion models is thus essential for a more realistic and holistic understanding of atmospheric pollution dynamics. This research addresses the limitations of traditional models by developing a mathematical framework that integrates periodic emission sources and temporally variable removal mechanisms. By doing so, we aim to capture the inherent time-dependent characteristics of both pollutant generation and attenuation processes. The resulting model leverages partial differential equations to describe the spatio-temporal evolution of pollutant concentration, incorporating parameters that reflect periodic forcing functions and dynamic removal terms.

The study also considers the role of advection by wind and diffusion driven by turbulence, offering a more comprehensive depiction of atmospheric pollutant behaviour [16,25,35]. Through analytical and numerical analysis, this model provides new insights into how periodic emissions and removal affect the dispersion patterns, peak concentrations, and potential accumulation of pollutants over time. Such a framework has valuable applications in environmental monitoring, policy formulation, and urban planning, especially in areas where pollution sources are non-continuous and highly variable. Ultimately, this work contributes to bridging the gap between theoretical modeling and real-world atmospheric conditions, paving the way for more accurate predictions and effective environmental interventions.

Mathematical Formulation: We consider a one-dimensional unsteady advection-diffusion-reaction model with the following governing equation:

$$\partial C/\partial t+u\ \partial C/\partial x=D\ (\partial^2 C)/(\partial x^2)-\lambda C+E(t)\delta(x).$$

where the pollutant concentration is denoted by $C(x,t)$, which varies with both spatial position x and time t. The transport of pollutants is influenced by a constant wind speed u, which drives advection, and a diffusion coefficient D, accounting for the spreading due to atmospheric turbulence. A key feature of the model is the inclusion of a removal mechanism characterized by a rate constant λ, which encapsulates processes such as chemical decay, dry or wet deposition, or gravitational settling that act to reduce the pollutant concentration over time. The source of pollution is modeled as a point source located at $x=0$, represented mathematically by the Dirac delta function $\delta(x)$. The emission rate from this source is not constant but periodic in nature, described by the function $E(t)=E0(1+\alpha\sin(\omega t))$ and $E0$ is the baseline (average) emission rate. The parameter α, with $0\leq \alpha <1$, quantifies the amplitude of the periodic modulation, determining the extent to which the emissions fluctuate over time. The angular frequency ω controls the rate of oscillation, corresponding to the frequency of periodic processes such as traffic cycles or industrial operations [3,15,31,36]. The full source term is therefore $E(t)\delta(x)$, capturing both the spatial localization and temporal variability of the emissions. This formulation allows the model to reflect real world scenarios more accurately by incorporating both dynamic emission patterns and removal effects within a unified framework.

Boundary Condition: To fully define the mathematical model and ensure the uniqueness and physical relevance of its solution, appropriate initial and boundary conditions are essential. The initial condition specified is $C(x,0)=0$, which assumes that at the initial time $t=0$, the atmosphere is free of any pollutant throughout the entire spatial domain [1,12,37]. This reflects a scenario where pollution begins only after the initiation of emissions from the point source and not from any pre-existing background contamination. Such an assumption is often adopted in modeling studies to isolate and examine the effect of the given emission source and its dynamic behavior over time. The spatial boundary conditions are given by $C(\pm\infty, t) = 0$, implying that far away from the source location, the pollutant concentration tends asymptotically to zero for all times $t \geq 0$. This condition is physically realistic and ensures that the domain is open and unbounded in both spatial directions, allowing the pollutant to disperse freely without artificial reflection or buildup at the domain boundaries [8, 26]. It also guarantees that the influence of the source diminishes with distance, which is consistent with the behavior of pollutant plumes in open atmospheric environments. The solution domain is defined as $x\in(-\infty, \infty)$ and $t \geq 0$, indicating that the pollutant can spread in both directions along the spatial axis and that the process is observed from the moment of emission onset into the future [7,24].

$C(x,0) = 0$ (initially no pollutant), (2)

$C(\pm\infty,t)=0$,

Solution domain: $x\in(-\infty,\infty), t\geq 0$.

Analytical Approach: To solve the PDE (1) , we use the method of Green's functions [11,27]. The Green's function for the advection-diffusion equation with decay and a point source is convolved with the time-dependent emission term:

$$C(x,t)=\int_0^t E(\tau)G(x,t-\tau)\,d\tau$$

where $G(x,t)$ is the fundamental solution of the advection-diffusion equation with decay:

$$G(x,t)=\frac{1}{\sqrt{4\pi Dt}} \exp\left(-\frac{(x-ut)^2}{4Dt}-\lambda t\right)$$

Substituting $E(t)$, we obtain:

$$C(x,t)=E_0 \int_0^t [1+\alpha\sin(\omega\tau)]\frac{1}{\sqrt{4\pi D(t-\tau)}} \exp\left[-\frac{(x-u(t-\tau))^2}{4D(t-\tau)}-\lambda(t-\tau)\right] d\tau$$

This expression represents the solution for $C(x,t)$, which must be evaluated numerically due to the complexity of the integrand. It accurately captures the combined effects of advection, diffusion, periodic emission, and exponential removal [7,28,38].

Numerical Results and Discussion: Air pollution remains one of the most pressing environmental and public health challenges worldwide, contributing to nearly 7 million premature deaths annually, primarily due to respiratory and cardiovascular diseases, according to the World Health Organization. Understanding the mechanisms that govern pollutant dispersion is therefore critical for developing effective mitigation strategies. In this study, a novel mathematical model was developed to investigate the dynamics of atmospheric pollutants subjected to periodic emission sources and constant atmospheric removal. The model incorporates realistic features such as oscillatory emission cycles and steady pollutant decay, enabling a comprehensive analysis of temporal and spatial variations in pollutant concentration [6, 14].

To explore the effects of key parameters on pollutant behavior, numerical simulations were conducted using MATLAB for different values of emission amplitude and removal rates, allowing visualization of concentration profiles over time and space. This approach not only provides insight into the underlying physical processes but also establishes a connection between pollutant dynamics and potential health outcomes, emphasizing the importance of accurate modeling in informing policy and public health interventions. Figure (2) shows the variation for pollutant concentration profile at different times. It shows the evolution of pollutant concentration profiles at different time snapshots. As time progresses, the pollutant plume moves downstream due to advection while spreading out due to diffusion. The influence of periodic emission is evident in the undulating pattern of concentration, and the amplitude of concentration decreases with time due to the removal effect.

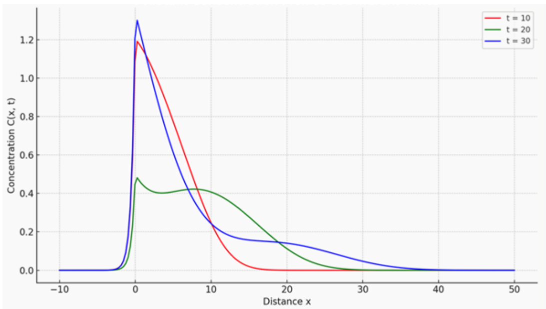

Figure (2): Pollutant Concentration Profiles at Different Times

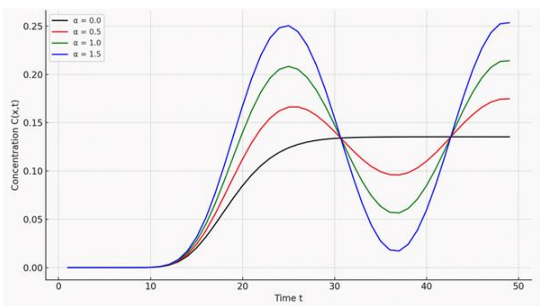

Figure (3): Effect of Emission Oscillation Amplitude (α) on Concentration Oscillation at Fixed Location

The results illustrate how the pollutant disperses over space and time, revealing key features of pollutant transport such as peak concentration movement and attenuation. Figure (3) shows the variation of emission amplitude parameter (α) on concentration oscillation at fixed location, x = 20. As increases, the oscillations in pollutant concentration become more pronounced, reflecting stronger periodicity in the source term. This demonstrates the sensitivity of concentration dynamics to periodic emission intensity [4,19,39]. For , the concentration grows smoothly, whereas larger values of induce significant temporal fluctuations. These results underline the importance of considering temporal variability in emission sources for accurate pollution forecasting. Figures illustrate how concentration profiles evolve over time and space under periodic emissions [12,32]. Temporal averaging over multiple cycles yields effective steady-state approximations useful for regulatory purposes. This model has direct applications in environmental monitoring, air quality forecasting, and urban planning. By capturing periodic behavior, it improves prediction accuracy for pollutant exposure and can inform time based emission control strategies.

II. CONCLUSION

A novel mathematical model incorporating periodic emissions and constant removal has been developed and analyzed. The combination of analytical and numerical approaches provides a comprehensive understanding of pollutant dynamics. Results show that higher values of the emission amplitude parameter lead to stronger oscillations in concentration over time, while increased removal rate results in faster decay of pollutants and lower overall concentration levels. These findings demonstrate the critical interplay between periodic emissions and atmospheric removal in shaping pollutant behavior. Importantly, given that the World Health Organization estimates that air pollution contributes to nearly 7 million premature deaths annually, primarily due to respiratory and cardiovascular diseases, this study highlights the significance of accurate pollutant modeling for informing effective mitigation strategies. Future work can extend this model to multi-dimensional and stochastic frameworks, further enhancing its relevance for protecting public health.

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