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Mathematical Formulation of Electromagnetic Radiation Using Vector Potential Expressions Derived from Maxwell's Equations

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Abstract- The electric vector potential F plays a significant role in electromagnetic field analysis when magnetic current sources are introduced through the equivalence principle. Although magnetic currents do not exist physically, their mathematical representation greatly simplifies the analysis of radiation, scattering, and aperture problems in electromagnetics and antenna theory. This work presents a detailed derivation of the electric vector potential F directly from Maxwell's equations by incorporating magnetic current density into the generalized field equations. Starting from the curl relations of Maxwell's equations, the electric field is expressed in terms of the curl of the electric vector potential, ensuring automatic satisfaction of Gauss's law in source-free regions. By applying appropriate vector identities and imposing a suitable gauge condition, a vector wave equation governing F is obtained. The solution of this wave equation leads to the retarded electric vector potential, which explicitly relates the magnetic current distribution to the radiated electromagnetic fields. The derived formulation provides physical insight into the radiation mechanism of equivalent magnetic current sources and establishes a dual framework to the conventional magnetic vector potential approach used for electric currents. The electric vector potential formulation is particularly advantageous for analyzing slot antennas, aperture radiation, electromagnetic scattering, and computational electromagnetics methods such as the Method of Moments and Finite Element Method. Overall, this derivation highlights the mathematical elegance and practical relevance of the electric vector potential in advanced electromagnetic radiation analysis.

Keywords: Electric Vector Potential, Magnetic Current Density, Maxwell's Equations and Electromagnetic Radiation.

I. INTRODUCTION

Electromagnetic radiation analysis is fundamentally governed by Maxwell's equations, which describe the behavior of electric and magnetic fields produced by time-varying sources. In conventional formulations, electric current density is treated as the primary source of radiation, and the associated magnetic vector potential is widely used to simplify field calculations. However, in many practical electromagnetic problems—such as aperture radiation, slot antennas, and scattering from openings the direct use of electric current sources becomes mathematically cumbersome. To address these challenges, magnetic current sources are introduced as equivalent sources using the equivalence principle, leading to an alternative but mathematically elegant representation of electromagnetic fields.

Within this dual framework, the electric vector potential F is employed to model the fields generated by magnetic current distributions. By expressing the electric field as the curl of the electric vector potential, Maxwell's equations can be systematically reduced to solvable wave equations under appropriate gauge conditions. This formulation not only provides deeper physical insight into radiation mechanisms but also offers significant computational advantages in analytical and numerical methods. The electric vector potential approach has become an essential tool in advanced electromagnetic theory, enabling accurate modeling of aperture and slot radiation, facilitating computational electromagnetics techniques, and supporting the design and analysis of modern antenna systems. Magnetic vector potential expression and its study are important for field analysis, which is detailed in [1]-[5]. The article presents an ultraweak discontinuous Petrov–Galerkin (DPG) formulation of the time-harmonic Maxwell equations for the vectorial envelope of the electromagnetic field in a weakly-guiding multi-mode fiber waveguide [6].

The paper, which investigate the modified characteristics finite element methods for the two- and three-dimensional incompressible magnetohydrodynamics system, formulated using a magnetic vector potential approach

[7]. A high-order method to evolve in time electromagnetic and velocity fields in conducting fluids with non-periodic boundaries is presented. The method has a small overhead compared with fast FFT-based pseudospectral methods in periodic domains [8]. Vector potential reconstruction from discrete magnetic field data enables the analytical evaluation of derivatives while ensuring solenoidal properties across the domain [9]. The E- and H-field distributions play a crucial role in the design and performance analysis of patch antennas detailed in [10]-[12]. These analysis improve the performance of antenna.

II. ANALYSIS

In electromagnetic theory, the magnetic vector potential A is a fundamental quantity used to describe magnetic fields generated by electric currents. Instead of working directly with the magnetic flux density B, it is often mathematically convenient to define B in terms of the vector potential as,

$$B = \nabla \times A \quad (1)$$

This definition automatically satisfies Gauss's law for magnetism,

$$\nabla . B = 0 \quad (2)$$

Since the divergence of a curl is always zero.

The magnetic vector potential arises naturally from Ampere–Maxwell's law. In magnetostatics, steady currents and time-invariant fields, Ampère's law is written as,

$$\nabla \times B = \mu J \quad (3)$$

Substitute the value of $B = \nabla \times A$ in the equation,

$$\nabla \times (\nabla \times A) = \mu J \quad (4)$$

But the vector identity,

$$\nabla \times (\nabla \times A) = \nabla(\nabla . A) - \nabla^2 A \quad (5)$$

From equation (4) substitute the value of $\nabla \times (\nabla \times A)$ in (5) produce that,

$$\mu J = \nabla(\nabla . A) - \nabla^2 A \quad (6)$$

To simplify this equation, a gauge condition is chosen. The most common choice is the Coulomb gauge,

$$\nabla . A = 0 \quad (7)$$

With this condition, the equation (6) reduces to a Poisson equation,

$$\mu J = 0 - \nabla^2 A \quad (8)$$

$$\nabla^2 A = -\mu J \quad (9)$$

Physically, the vector potential represents the spatial distribution of magnetic effects due to current flow. While A itself is not directly measurable, it plays a crucial role in calculating observable quantities such as, $B = \nabla \times A$. The electric field intensity is expressed as

$$E = -\frac{\partial A}{\partial t} - \nabla \emptyset \quad (10)$$

In antenna theory, the vector potential is a powerful tool for deriving radiation fields, input impedance, and far-field expressions.

In classical electromagnetics, magnetic current sources do not exist physically in the same way as electric currents. However, they are extremely useful mathematical constructs introduced through the equivalence principle, especially in antenna theory, aperture radiation, and computational electromagnetics. A magnetic current source is usually denoted by M(magnetic current density), though in some texts it is loosely written as J_m. To analyse fields due to magnetic currents, electromagnetic theory introduces dual vector potentials, complementing the electric current–based formulation.

For an electric current source J, the magnetic vector potential A is defined such that,

$$B = \nabla \times A$$

By electromagnetic duality, when a magnetic current source M is present, an electric vector potential F is introduced such that:

$$E = -\nabla \times F \quad (11)$$

However, in generalized formulations (especially in radiation problems), both electric and magnetic current sources may coexist, and the fields are expressed using both vector potentials A and F.

Maxwell's equations with magnetic current sources, including magnetic current density M Maxwell's curl equations become:

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} - M \quad (12)$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad (13)$$

Substitute the value $D = \varepsilon_0 E$ in the Ampere's equation,

$$\nabla \times H = J + \varepsilon_0 \frac{\partial E}{\partial t} \quad (14)$$

For radiation problems involving only magnetic current source ($J = 0$), the electric vector potential F becomes dominant, but the magnetic vector potential A can still be defined through duality relations.

For time-varying magnetic current density $M(r, t)$ the electric vector potential F satisfies the wave equation can be derived by using Maxwell's equations, the equation (11) and (12) are,

$$E = -\nabla \times F \quad (11)$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} - M \quad (12)$$

Substitute the value of E in (12)

$$\nabla \times (-\nabla \times F) = -\mu \frac{\partial H}{\partial t} - M \quad (15)$$

Using the vector identity,

$$\nabla \times (\nabla \times F) = \nabla(\nabla \cdot F) - \nabla^2 F \quad (16)$$

We can rewrite the equation (13) by using (14),

$$-\nabla(\nabla \cdot F) + \nabla^2 F = -\mu \frac{\partial H}{\partial t} - M \quad (17)$$

But the Ampere's law (14), with $J = 0$,

$$\nabla \times H = \varepsilon_0 \frac{\partial E}{\partial t} \quad (18)$$

Substitute the value of E from (11), $E = -\nabla \times F$ and the equation (18) become,

$$\nabla \times H = \varepsilon_0 \frac{\partial}{\partial t} (-\nabla \times F\,) \quad (19)$$

$$\nabla \times H = \nabla \times \varepsilon_0 \frac{-\partial F}{\partial t} \quad (20)$$

The equation (20) implies that,

$$H = \varepsilon_0 \frac{-\partial F}{\partial t} \quad (21)$$

Substitute the value of H in equation (17),

$$-\nabla(\nabla \cdot F) + \nabla^2 F = -\mu \frac{\partial \left(\varepsilon_0 \frac{-\partial F}{\partial t}\right)}{\partial t} - M \quad (22)$$

$$-\nabla(\nabla \cdot F) + \nabla^2 F = \mu \varepsilon_0 \frac{\partial^2 F}{\partial t^2} - M \quad (23)$$

$$\nabla^2 F - \mu \varepsilon_0 \frac{\partial^2 F}{\partial t^2} = \nabla(\nabla \cdot F) - M \quad (24)$$

Which is the wave equation for F.

The electric vector potential F is derived directly from Maxwell's equations when magnetic current sources are introduced. By defining E as the curl of F and applying an appropriate gauge, a clean wave equation is obtained whose solution describes electromagnetic radiation from equivalent magnetic current sources. This formulation is essential for slot antennas, aperture radiation, scattering analysis, and computational electromagnetics.

The electric field (E-field) distribution plot represents the spatial variation of the electric field intensity and direction in a given electromagnetic structure or region. It illustrates how electric energy is stored and distributed due to applied sources or boundary conditions. In radiation and antenna problems, the E-field is closely associated with voltage variation and charge accumulation, particularly near conducting surfaces, edges, and apertures.

The magnetic field (H-field) distribution plot shows the spatial variation of magnetic field intensity generated by current flow. Unlike the electric field, the H-field is directly associated with current distribution, making it particularly useful for analyzing current paths and magnetic coupling. Regions of strong H-field intensity typically appear around conductors, current paths, and loops, following the right-hand rule. In antennas, the H-field is maximum where surface currents are strongest. For example, in microstrip patch antennas, the H-field is concentrated beneath the patch and along the feed region, while in slot antennas it encircles the slot opening. A smooth and continuous H-field pattern generally indicates efficient current flow and proper excitation of the intended mode. The E and H field of microstrip patch antenna is shown in the Fig.1. Based on this plot, the proposed antenna is simulated and fabricated, which is detailed in Fig. 2.

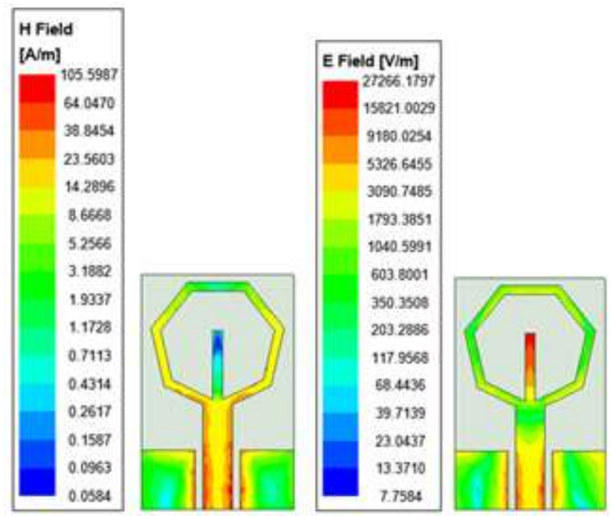

Fig. 1 E and H field distribution plot of patch antenna.

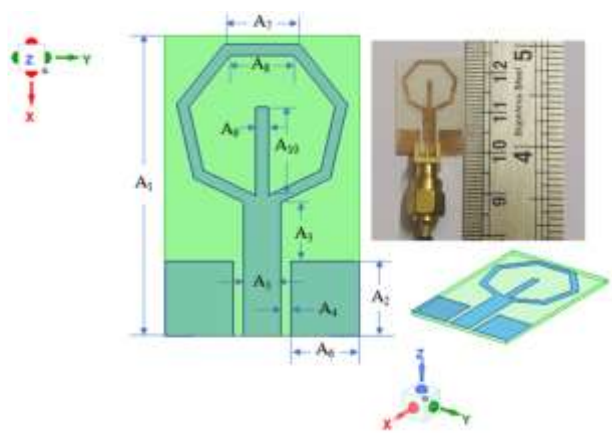

Fig. 2 proposed simulated and fabricated antenna.

III. CONCLUSION

This work has presented a detailed theoretical derivation of the electric vector potential F based on Maxwell's equations with the inclusion of magnetic current sources. By employing the equivalence principle and electromagnetic duality, the electric field was expressed in terms of the curl of the electric vector potential, allowing Maxwell's equations to be systematically reduced to a vector wave equation. The application of an appropriate gauge condition led to a compact and physically meaningful formulation, whose solution in the form of a retarded potential establishes a direct relationship between magnetic current distributions and the resulting electromagnetic fields. This derivation provides a rigorous mathematical foundation for understanding radiation mechanisms associated with equivalent magnetic current sources.

The electric vector potential formulation offers significant advantages in both analytical and computational electromagnetic analysis. It enables efficient modelling of aperture and slot antennas, simplifies radiation and scattering problems, and supports numerical techniques such as the Method of Moments and Finite Element Method. Beyond its mathematical elegance, the F potential framework enhances physical insight into electromagnetic radiation phenomena and serves as a powerful dual complement to the conventional magnetic vector potential approach. Consequently, the electric vector potential remains an essential tool in advanced electromagnetic theory and modern antenna engineering applications.

REFERENCE

1. Song, H., Xue, Y. & Yan, C. Three-dimensional unstructured adaptive finite element forward modeling simulation of magnetotellurics based on magnetic vector potential-electric scalar potential. Environ Earth Sci 83, 494 (2024). https://doi.org/10.1007/s12665-024-11802-z
2. Rosen, I.T., Muschinske, S., Barrett, C.N. et al. A synthetic magnetic vector potential in a 2D superconducting qubit array. Nat. Phys. 20, 1881–1887 (2024).
3. Tian, S., Yao, Z., Wygant, J.R. et al. Evidence for Alfvén waves powering auroral arc via a static electric potential drop. Nat Commun 17, 297 (2026). https:/ /doi.org/10.1038/s41467-025-65819-4
4. Ding, Q., Long, X., Mao, S. et al. Second Order Unconditionally Convergent Fully Discrete Scheme for Incompressible Vector Potential MHD System. J Sci Comput 100, 1 (2024). https://doi.org/10.1007/s10915-024-02553-x
5. Blaszczyk, M., Hackl, K. A novel class of electro-mechanical metamaterials for stress reduction through electric fields. Continuum Mech. Thermodyn. 37, 53 (2025). https://doi.org/10.1007/s00161-025-01385-w
6. Henneking, Stefan, Jacob Grosek, and Leszek Demkowicz. "A vectorial envelope Maxwell formulation for electromagnetic waveguides with application to nonlinear fiber optics." Computers & Mathematics with Applications 193 (2025): 34-53.
7. Si, Zhiyong, Puxuan Xu, Yunxia Wang, and Hongwei Wang. "The modified characteristics FEMs for the incompressible vector potential MagnetoHydroDynamics system." Communications in Nonlinear Science and Numerical Simulation (2025): 109150.
8. Fontana, Mauro, Pablo D. Mininni, Oscar P. Bruno, and Pablo Dmitruk. "Vector potential-based MHD solver for non-periodic flows using Fourier continuation expansions." Computer Physics Communications 275 (2022): 108304.
9. Beznosov, Oleksii, Jesus Bonilla, Xianzhu Tang, and Golo Wimmer. "High order interpolation of magnetic fields with vector potential reconstruction for particle simulations." arXiv preprint arXiv:2501.01523 (2025).
10. Vasu, Abhilash S., Lakshmi N. R, Aswathi B, and Dana S. "Hybrid Nanocomposite Graphene-TiO_2 and Graphene-ZnO Coated Stub Enhance the Electromagnetic Radiation of Flexible Radiator." Brazilian Journal of Physics 56, no. 2 (2026): 62.
11. Vasu, A. S., N. R. Lakshmi, and T. K. Sreeja. "Nanocomposite-filled slot enhancing radiation of a triple band antenna." Russian Physics Journal (2025): 1-9.
12. Vasu, Abhilash S., N. R. Lakshmi, and T. K. Sreeja. "A compact circular slotted radiator for ISM and wireless bands." Russian Physics Journal (2025): 1-7.