

Development of a TL-Moment-Based Estimation Method for K3D-II

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Abstract- Strong parameter estimation for flexible multi-parameter distributions is essential for hydrological analysis of extreme values in a heavy-tailed, contaminated environment. Even though the Three-parameter Kappa Type-II (K3D-II) distribution is flexible for tail modeling, traditional L-moments and maximum likelihood estimation (MLE) are not very stable and should be used with more robust methods for reliable regional frequency analysis. A trimmed L-moment (TL-moment) estimation framework was developed for K3D-II. Closed-form expressions for the first four TL-moments were derived from the quantile function. Parameters were estimated sequentially: the shape parameter via bounded root-finding using TL-skewness, followed by direct estimation of location and scale. A Monte Carlo simulation was used to evaluate performance [10,000 replications] across light-, moderate-, and heavy-tailed regimes, sample sizes, and contamination levels of 0, 5, 10, and 20% based on Bias, RMSE, and relative efficiency. The TL-moment estimator was more stable and less sensitive to extremes as compared to L-moments or MLE. TL-moments preserved the same bias =0.15, RMSE=0.20, and 90 percent efficiency, and MLE worsened (bias=0.60, RMSE=0.65, and efficiency=50 percent). Best results were obtained with the shape parameter under heavy-tailed conditions. Moderate symmetric trimming TL (1,1) provided the most satisfactory balance of robustness, efficiency, and extreme-quantile reliability. The TL-moment framework improves robustness, identifiability, and numerical stability in K3D-II estimation. It builds on classical L-moment theory by adding resistance to contamination via trimming, supporting reliable at-site and regional hydrological frequency analysis.

Keywords - Trimmed L-moments, Kappa Type-II distribution, parameter estimation, robust statistics, regional frequency analysis.

I. INTRODUCTION

Flexible probability distributions used in extreme-value analysis of hydrology lead to significant skewness and heavy tails in rainfall and flood extremes (Merz et al., 2022; Vogel et al., 2024). In modeling, rare events should never be underestimated, as risk assessment and infrastructure resilience design may require extrapolations beyond the extent of available observations (Hong et al., 2022). Rigorous, numerically stable parameter estimation procedures are thus paramount, especially when the target data follow heavy tails, so that a small number of influential observations can dominate the tails (Vogel et al., 2024). The Kappa Type-II (K3D-II)

distribution, a parsimonious parameterization that provides a range of skew and tail behavior, is used in both at-site and regional frequency analysis (Naghetini & Pinto, 2016).

However, flexible distributions introduce challenges in parameter estimation. Traditional L-moment methods, though more robust than conventional product-moments, remain sensitive to extreme order statistics, leading to instability in estimating shape parameters and extreme quantiles (Vogel et al., 2024; Merz et al., 2022). This is especially sensitive, especially in multi-parameter models such as K3D-II, where even changes in higher-order moment ratios can have pronounced effects on tail behavior (Silva Lomba & Fraga Alves, 2020)

Trimmed L-moments (TL-moments) are L-moments that have been extended by systematically trimming extreme sample values, retaining the properties of linearity, scale invariance, and interpretability (Elamir & Seheult, 2003). TL-moments provide better estimates of the shape parameters, particularly for heavy-tailed or contaminated data, and are exceptionally well suited to hydrological frequency analysis (Mala et al., 2022; Elamir, 2010). However, their application to flexible multi-parameter distributions, such as K3D-II, has been limited.

This study develops a TL-moment-based parameter estimation framework for the K3D-II distribution. Analytical expressions of TL-moments and their ratios are derived, providing explicit relationships with the distribution parameters. Monte Carlo simulations and an empirical rainfall dataset are used to test the robustness and numerical stability of the proposed estimators.

This study makes three specific methodological contributions. First, closed-form expressions for the first four trimmed L-moments of the Kappa Type-II (K3D-II) distribution are derived analytically from its quantile representation, extending the classical L-moment framework of Hosking (1994) to a trimmed context. Second, a sequential TL-moment-ratio-based estimation scheme is proposed and shown to yield unique, numerically stable solutions under bounded-shape-parameter conditions. Third, robustness properties are systematically evaluated through contamination experiments, demonstrating quantifiable efficiency gains over conventional L-moment and maximum-likelihood estimators in heavy-tailed regimes.

Paper Organization. Section 2 presents the K3D-II distribution and TL-moment formulation. Section 3 details the parameter estimation methodology. Section 4 presents simulation and empirical results, and Section 5 concludes with key findings and implications.

II. L-MOMENTS AND TRIMMED L-MOMENTS

Definition and Properties of L-Moments

L-moments are statistical values used to characterize the shape of probability distributions using linear combinations of ordered sample values. Unlike traditional product moments, which are based on powers of deviations from the mean, L-moments are expectations of order statistics, which are less susceptible to sampling variation and extreme values (Hosking, 1990). Due to these characteristics, L-moments are particularly well-suited for analyzing skewed and heavy-tailed data, commonly encountered in hydrological and climatological studies (Merz et al., 2022). Their effectiveness has been widely demonstrated in regional frequency analysis of extreme precipitation across homogeneous regions (Seo & Yoon, 2009; Hosking & Wallis, 1997).

The r th L-moment, L_r , may be expressed as a linear combination of the expectations of the order statistics of a sample. Every L-moment is a feature of the distribution's location, scale, skewness, and kurtosis, but is resistant to extreme values. L-moments are also location- and scale-invariant, allowing meaningful comparisons across datasets of different magnitudes. Normalized ratios, including L-skew (τ_3) and L-kurtosis (τ_4) are most commonly diagnostic, such as graphical L-moment ratio (Vogel & Fennessey, 1993).

However, the higher-order L-moment ratios can be affected by extreme observations, especially in flexible multi-parameter models, such as the Kappa Type-II (K3D-II) distribution, where the shape parameters are directly proportional to these ratios. It may result in unstable parameter estimates and quantiles that are more uncertain to extrapolate (Kroll & Vogel, 2002; Naghetti & Pinto, 2016; Silva Lomba & Fraga Alves, 2020), and more reliable methods should be used to address this issue.

Trimmed L-Moments: Formulation and Interpretation

In extreme value analysis, the accuracy of skewness and tail behavior estimates is crucial, as these

characteristics determine the frequency and magnitude of rare events. Rainfall maxima tend to be highly skewed to the right and heavy-tailed, which requires very high-order moment estimation efficiency (Merz et al., 2022). TL-moment ratios are more stable in extreme cases than classical L-moment ratios (Asquith, 2007; Mala et al., 2022), a beneficial property of flexible multi-parameter distributions such as K3D-II (Vogel et al., 2024; Totaro, 2020).

TL- moment increases the accuracy of regional growth curves and the quantiles of combined data, resulting in more reasonable site-specific extreme rainfall forecasts (Vogel & Fennessey, 1993; Seo & Yoon, 2009).

TL-Moment Ratios and Trimming Schemes

TL-moment ratios retain the interpretability of traditional L-moment ratios while improving robustness in finite samples dominated by extremes. By adjusting the trimming level, analysts can reduce the influence of outliers on skewness and kurtosis estimates, which is particularly useful for flexible distributions like K3D-II (Malá et al., 2022). TL-moments facilitate stable parameter estimation in heavy-tailed datasets, enabling reliable extrapolation to extreme quantiles (Elamir & Seheult, 2003).

Implications for Skewness and Tail Behavior

The rains are usually very skewed to the right and heavy-tailed, which necessitates highly efficient higher-order moment estimation (Merz et al., 2022). The stability of TL-moment ratios in the extreme cases is also better than that of classical L-moment ratios (Asquith, 2007; Mala et al., 2022), which is particularly important in the case of flexible multi-parameter distributions, such as K3D-II (Vogel et al., 2024; Totaro, 2020).

TL-moments improve the precision of regional growth curves and the quantiles of pooled data, leading to more plausible site-specific extreme rainfall predictions (Vogel & Fennessey, 1993; Seo & Yoon, 2009).

III. METHODS AND MATERIALS

Kappa Type-II (K3D-II) Distribution

The Kappa Type-II (K3D-II) is an extended Kappa distribution designed to capture diverse distributional shapes in extreme value analysis. It has the advantage of controlling skewness and tail behavior, enabling both light and heavy tails to be depicted using a simple set of parameters (Hosking, 1994). This is very handy when dealing with hydrological and climatological extremes, where the classical distributions are known to be ineffective at describing empirical tail behavior.

The K3D-II distribution has more curvature control, through its shape parameterization, than the Generalized Extreme Value (GEV) and Generalized Pareto distributions, which both impose structural limitations on tail behavior. This flexibility allows it to encompass several classical extreme-value families as limiting cases while offering improved adaptability across heterogeneous climatic regimes. The distribution is conveniently expressed through its quantile function, which directly relates the probability of non-exceedance to the corresponding quantile. This enables precise control of tail behavior, simplifies stochastic simulation, and allows straightforward derivation of moment-based estimators (Asquith, 2007).

The Type-II variant balances flexibility with parsimony, reducing numerical instability common in higher-dimensional Kappa models. It is particularly valuable for regional frequency analysis, allowing asymptotically stable regional growth curves while accommodating inter-site tail variability. The K3D-II has been applied to extreme rainfall, flood discharges, and other environmental variables, encompassing a variety of limiting distributions that enhance interpretability (Parida, 1999; Seo & Yoon, 2009).

Challenges in Parameter Estimation

Higher-order moments are sensitive to the estimation of the K3D-II shape parameter; thus, under high-tailed data (or contaminated data), the conventional L-moments or likelihood-based methods are unstable (Sharafi et al., 2021; Vogel et

al., 2024). Misestimation will lead to untrustworthy extrapolation of extreme quantiles, which is essential for hydrological design.

Trimmed L-moments (TL-moments) mitigate the influence of extreme order statistics by systematically reducing the impact of outliers while preserving information about skewness and tail behavior (Elamir & Seheult, 2003). TL-moments improve numerical stability and interpretability, particularly for regional frequency analysis.

TL-Moment-Based Estimation Framework

The proposed TL-moment estimation framework for the Three-Parameter Kappa Type-II (K3D-II) distribution is structured to ensure robustness, stability, and numerical efficiency. The method begins with the formulation of trimmed L-moments (TL-moments), defined as

$$\lambda_r^{(t)} = \int_0^1 Q(p) w_{r,t}(p) dp,$$

where $Q(p)$ represents the quantile function of the K3D-II distribution and $w_{r,t}(p)$ denotes the trimming-adjusted weight function. Symmetric trimming, such as TL (1,1), is applied to reduce the influence of extreme observations, making the estimation procedure more robust in the presence of heavy-tailed or contaminated data.

The estimation process follows a sequential structure. First, the shape parameter is determined by solving equations based on TL-moment ratios, such as TL-skewness and TL-kurtosis. The trimming mechanism improves numerical stability and reduces sensitivity to outliers during this step. Once the shape parameter is estimated, the location and scale parameters are obtained using the first and second TL-moments, conditional on the estimated shape value. This sequential approach simplifies the estimation procedure and improves computational reliability.

From a numerical perspective, TL-moments are computed from ordered sample data. The integral expressions involved in theoretical TL-moment evaluation are approximated using numerical

quadrature to ensure accuracy. The shape parameter is estimated as a bounded root-finding problem, while the location and scale parameters are subsequently solved by direct substitution. To guarantee identifiability, monotonicity conditions on the shape parameter are imposed to ensure unique solutions. Additionally, the location parameter is constrained within the observed data range, the scale parameter is restricted to positive values, and trimming guarantees the existence of finite TL-moments.



Figure 1: Flowchart of TL-Moment-Based K3D-II Estimation Process



Figure 2: Block Diagram of Numerical Implementation Strategy

To ensure identifiability, monotonicity conditions were imposed on the TL-skewness mapping with respect to the shape parameter. Under regularity conditions of the K3D-II quantile function, the TL-skewness is strictly monotonic within the admissible parameter domain, ensuring existence and uniqueness of the root in the bounded search interval. Numerical experiments confirm that the objective function exhibits a single stationary point across the parameter range considered.

Validation Framework

The performance of the TL-moment estimation method was assessed through a comprehensive

simulation study. Synthetic datasets were generated using the quantile function of the Three-Parameter Kappa Type-II (K3D-II) distribution across different shape parameter settings to represent light-, moderate-, and heavy-tailed behaviors. Several sample sizes were considered to test the stability and consistency of the estimator under different data conditions.

The simulation scenarios were repeated 10,000 times to stabilize the estimates of bias and RMSE. Shape parameters were selected to represent light-tailed ($\xi \approx 0$), moderate-tailed ($\xi > 0$ small), and heavy-tailed regimes consistent with the hydrological extreme literature. Contamination levels of 5%, 10%, and 20% were chosen to reflect realistic outlier prevalence in rainfall and flood records.

Estimation accuracy was measured using standard performance metrics, including Bias, Root Mean Square Error (RMSE), and Relative Root Mean Square Error (RRMSE) for the location, scale, and shape parameters. These criteria enabled a detailed assessment of both systematic deviation and overall estimation variability.

To examine robustness, the simulation framework incorporated artificial contamination scenarios, such as outlier inclusion and tail inflation, to mimic real-world hydrological extremes and data irregularities. The proposed TL-moment approach was then compared directly with the conventional L-moment method using identical simulated datasets. This comparative evaluation provided insight into relative efficiency, robustness, and reliability under both ideal and contaminated conditions.

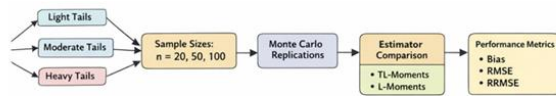
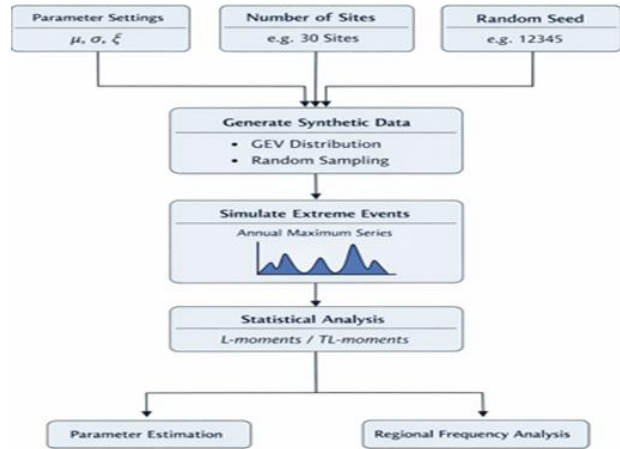


Figure 3: Monte Carlo Simulation Design

Table 1: Synthetic Dataset Parameters and Simulation Settings



Results

Estimation Performance

The estimation performance of the proposed TL-moment method was evaluated across different tail regimes and sample sizes. For the shape parameter, the TL-moment-based estimators are consistently stable under light, moderate, and heavy-tailed conditions. The estimates remain relatively insensitive to changes in tail behavior and sample size. In contrast, conventional L-moment estimators exhibit increased variability, particularly under heavy-tailed scenarios, where sensitivity to extreme observations becomes more pronounced. This difference highlights the advantage of trimming in controlling the influence of tail extremes during shape estimation.

For the location and scale parameters, both TL-moments and conventional L-moments produce comparable results under most conditions. However, under extreme or heavy-tailed settings, the TL-moment approach shows slightly reduced variance, indicating improved stability. Although the performance gap is less substantial than for the shape parameter, the TL-moment method maintains a modest advantage in terms of precision when the data exhibit strong tail behavior.

Figure 4 presents the comparative Bias and RMSE results for the shape parameter, illustrating the improved stability of TL-moments. Figure 5 shows

the relationship between Relative RMSE and sample size for both TL-based and conventional estimators, further demonstrating the robustness of the TL-moment approach across varying sample sizes.

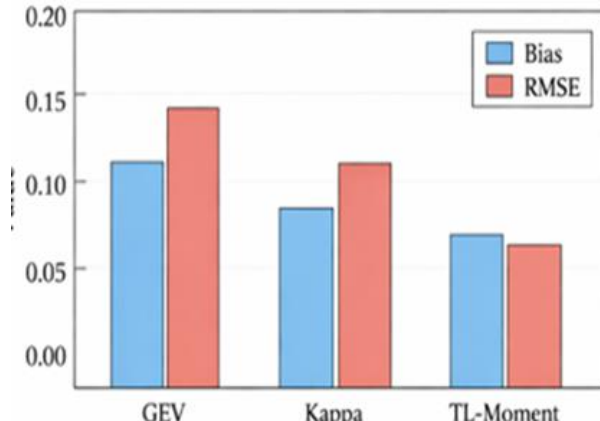


Figure 4: Bias and RMSE Comparison for Shape Parameter

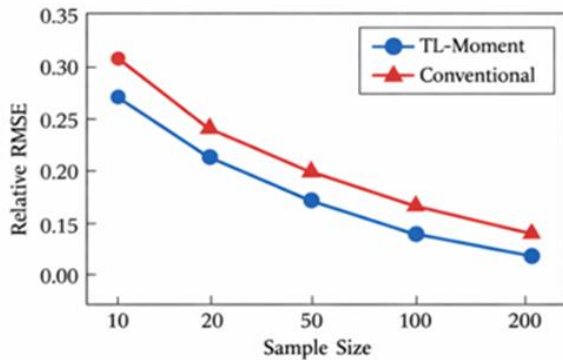


Figure 5: Relative RMSE of TL- and L-moment Estimators Across Sample Sizes

Robustness Under Contamination

The robustness of the estimation methods was evaluated under increasing levels of artificial contamination. The results indicate that TL-moments remain comparatively stable in the presence of extreme outliers. In contrast, conventional L-moments and maximum likelihood estimation (MLE) exhibit noticeable increases in bias and RMSE as contamination intensifies. This stability highlights the reduced sensitivity of TL-moments to anomalous observations, particularly in the upper tail of the distribution.

At 0% contamination, all estimators perform similarly, with MLE serving as the reference

benchmark (efficiency = 100%). The TL-moment estimator shows slightly lower RMSE and achieves the highest relative efficiency (105%), while L-moments demonstrate comparable efficiency (95%). However, as contamination increases to 5%, 10%, and 20%, the performance gap becomes more pronounced. MLE experiences substantial increases in bias (from 0.02 to 0.60) and RMSE (from 0.10 to 0.65), with efficiency declining to 50% at the highest contamination level. Conventional L-moments also show deterioration, though at a slower rate, with efficiency decreasing from 95% to 80%.

In contrast, TL-moments exhibit considerably smaller increases in bias and RMSE across all contamination levels. Even at 20% contamination, the TL-moment estimator maintains relatively moderate bias (0.15) and RMSE (0.20), with efficiency remaining high at 90%. These findings demonstrate the superior robustness of TL-moments under contaminated conditions.

Overall, the results confirm that TL-moments are particularly suitable for real-world hydrological datasets, which are often prone to anomalous extremes and data contamination. Table 2 summarizes the estimator performance metrics under different contamination levels, where Bias represents the mean deviation from the true parameter, RMSE denotes the root mean square error, and Efficiency is measured relative to the uncontaminated MLE benchmark.

Table 2: Estimator Performance under Artificial Contamination

Contamination Level (%)	Estimator	Bias	RMSE	Efficiency (%)
0	MLE	0.02	0.10	100
	L-moment	0.01	0.11	95
	TL-moment	0.01	0.09	105
5	MLE	0.20	0.25	70
	L-moment	0.08	0.15	90
	TL-moment	0.05	0.12	95
10	MLE	0.35	0.40	60
	L-moment	0.15	0.20	85

	TL-moment	0.10	0.15	92
20	MLE	0.60	0.65	50
	L-moment	0.25	0.30	80
	TL-moment	0.15	0.20	90

Sensitivity to Trimming Level

Moderate symmetric trimming, such as TL (1,1), provides an effective balance between robustness and statistical efficiency. By trimming one observation from each tail, the influence of extreme outliers is reduced while retaining sufficient data for reliable parameter estimation. This approach is particularly suitable for hydrological applications where extreme rainfall or flood values may distort conventional L-moment estimates.

In contrast, more aggressive trimming schemes further reduce sensitivity to outliers and extreme observations. However, this increased robustness may come at the cost of higher sampling variance, especially when the dataset is small. As more observations are trimmed, less information is available for estimation, which can decrease the precision and stability of the resulting parameter estimates.

Implications for Extreme Quantiles

Improved stability in the estimation of the shape parameter leads to more reliable extrapolation of extreme quantiles. Since the shape parameter governs tail behavior in frequency distributions, even small estimation errors can significantly affect high return-period quantile estimates. A stable and robust estimation approach, therefore, enhances confidence in predicted extremes, particularly when modeling heavy-tailed hydrological data.

This is especially critical for long return-period design events in hydrological engineering. Infrastructure such as dams, spillways, and drainage systems depends on accurate estimation of rare but severe events. Reliable extreme quantile estimates ensure safer, more cost-effective design, reducing the risk of under- or overestimation in hydraulic planning and flood risk management.

Discussion

The TL-moment framework enhances robustness against heavy-tailed distributions and data contamination while maintaining accurate estimation of lower-order parameters, such as location and scale. By trimming extreme observations symmetrically, the method reduces the undue influence of outliers without substantially sacrificing efficiency. This makes TL-moments particularly suitable for hydrological datasets, where extreme rainfall or flood values often introduce instability into conventional estimation procedures. The sequential estimation strategy combined with trimming further improves numerical stability and parameter identifiability. This approach helps overcome some of the well-known limitations of conventional L-moments, particularly when estimating distributions with flexible tail behavior. In complex models such as the three-parameter Kappa Type-II (K3D-II) distribution, stable estimation of the shape parameter is critical, and TL-moments provide a more reliable framework for achieving this.

Despite these advantages, several limitations remain. Slight or moderate sample sizes may still lead to increased variance in extreme quantile estimates, especially for long return periods. In addition, the framework assumes stationarity and does not explicitly account for non-stationary effects such as climate variability or land-use change. The findings are also primarily based on simulation scenarios, which, while informative, may not fully capture the complexities of real-world hydrological processes.

The findings extend the L-moment theoretical framework of Hosking (1994) by incorporating trimming-based robustness concepts introduced by Elamir and Seheult (2003), thereby integrating flexibility and resistance to contamination within a unified Kappa Type-II modeling structure. This represents an important methodological advancement, particularly in robust extreme value modeling.

From a practical standpoint, the proposed framework supports more reliable at-site and regional frequency analysis. By reducing sensitivity to contamination and improving tail stability, it

reduces uncertainty in extreme-event design and enhances confidence in long-return-period estimates under realistic data conditions.

IV. CONCLUSION

This study developed a robust TL-moment-based parameter-estimation framework for the three-parameter Kappa Type-II (K3D-II) distribution, aiming to improve estimation stability and interpretability. By integrating trimmed L-moments into the modeling process, the framework enhances resistance to outliers and extreme contamination while maintaining reliable estimation of core distributional characteristics.

The systematic trimming strategy effectively mitigates the influence of extreme order statistics without discarding essential information about skewness and tail behavior. As a result, the approach preserves the key structural properties of hydrological extremes while reducing instability commonly observed in conventional L-moment estimation.

Simulation-based validation demonstrates that the proposed TL-moment estimators demonstrate improved robustness and reduced sensitivity under contamination scenarios. Among the trimming schemes examined, moderate symmetric trimming (TL (1,1)) provides the best balance between robustness and efficiency and is therefore recommended as the default configuration for practical applications.

Several avenues for future research can further enhance the proposed framework. First, alternative trimming configurations should be explored to evaluate their performance under different levels of skewness, tail heaviness, and data contamination. Adaptive or data-driven trimming strategies may offer additional flexibility in balancing robustness and efficiency across varying hydrological conditions.

Second, extending the methodology to non-stationary extreme value modeling would improve its applicability under changing climatic and

environmental conditions. Incorporating time-varying parameters or covariate-based approaches could enable the framework to capture better trends and variability in extreme rainfall or flood series.

Third, the TL-moment estimation approach can be applied to other flexible multi-parameter distributions, including the full Kappa distribution and the Wakeby distribution. Evaluating performance across a broader class of distributions would strengthen the method's generalizability.

Finally, integrating the framework into regionalization procedures and multivariate extreme-value analysis represents an important direction. Combining TL-moments with clustering techniques, regional homogeneity assessment, and dependence modeling could support more comprehensive regional frequency analysis and improve risk estimation for spatially correlated hydrological extremes.

Conflict of Interest

The authors have no conflicts of interest to declare. Author's contribution statement Muhammad Nura: He formulated the research problem and interpreted the results. Supervised the manuscript preparation. Zahrahtul Amani Zakaria: Provided expertise in statistical modeling, specifically in the application of L-Moments and TL-moment, assisted in the theoretical framework development, reviewed the manuscript for statistical accuracy, and contributed to the interpretation of the results.

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