

Quantum Computing for Combinatorial Optimization: Algorithms, Complexity Analysis, and Real-World Applications.

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Abstract- Combinatorial optimization problems play a vital role in computer science and engineering, with applications in logistics, network design, finance, and resource allocation. However, many of these problems are NP-hard, making them computationally expensive to solve using classical optimization techniques, especially for large-scale instances. In recent years, quantum computing has emerged as a promising paradigm capable of addressing these limitations by leveraging quantum mechanical principles such as superposition and entanglement. This study investigates the application of quantum computing for solving combinatorial optimization problems, with a focus on algorithm design, complexity analysis, and real-world applicability. The research primarily employs the Quantum Approximate Optimization Algorithm (QAOA) and Grover's search algorithm to solve representative problems such as Max-Cut and the Knapsack problem. The performance of these algorithms is evaluated using key metrics including accuracy, execution time, and approximation ratio, and is compared with classical optimization techniques. The results demonstrate that QAOA is capable of generating near-optimal solutions, achieving accuracy levels of up to 93% for small problem instances. However, the study also highlights the challenges associated with current quantum systems, including noise, limited qubit availability, and increased execution time. Comparative analysis reveals that while classical methods remain more efficient for small-scale problems, quantum algorithms show significant potential for scalability and handling complex optimization tasks. The findings suggest that hybrid quantum-classical approaches offer a practical solution in the current Noisy Intermediate-Scale Quantum (NISQ) era. The study concludes that although quantum computing is still evolving, it holds strong potential for transforming combinatorial optimization in the future.

Keywords: Quantum Computing, Combinatorial Optimization, Quantum Approximate Optimization Algorithm (QAOA), Grover's Algorithm, Max-Cut Problem, Knapsack Problem, NISQ, Hybrid Quantum-Classical Models, Optimization Algorithms, Computational Complexity.

I. INTRODUCTION

Combinatorial optimization plays a critical role in computer science and engineering, addressing complex decision-making problems across domains such as logistics, transportation, finance, and network design. These problems, including the Traveling Salesman Problem (TSP), Knapsack Problem, and graph partitioning, are typically classified as NP-hard, meaning that the computational effort required to obtain optimal solutions increases exponentially with problem size. Traditional optimization techniques, although effective for small-scale instances, often struggle to provide efficient solutions for large and dynamic

problem environments (Jiménez et al., 2025; Gharehchopogh, 2023).

Over the past few decades, classical approaches such as exact algorithms, heuristics, and metaheuristics have been extensively developed to tackle combinatorial optimization problems. Techniques like genetic algorithms, simulated annealing, and evolutionary computation have demonstrated significant success in approximating near-optimal solutions. However, these methods are limited by issues such as local optima convergence, scalability constraints, and high computational complexity (Ur Rehman et al., 2025). Furthermore, as data-intensive applications continue to grow, the demand for more

efficient and scalable optimization techniques has increased substantially.

In this context, quantum computing has emerged as a promising paradigm capable of addressing the limitations of classical optimization methods. By leveraging quantum mechanical principles such as superposition, entanglement, and interference, quantum computers have the potential to explore solution spaces more efficiently than classical systems. Recent advancements in quantum algorithms, particularly the Quantum Approximate Optimization Algorithm (QAOA), quantum annealing, and Grover-based search techniques, have opened new avenues for solving combinatorial optimization problems (Blekos et al., 2024; Boulebnane & Montanaro, 2024).

Despite these theoretical advantages, practical implementation of quantum optimization remains challenging. Current quantum devices operate in the Noisy Intermediate-Scale Quantum (NISQ) era, characterized by limited qubit counts, noise, and decoherence, which restrict their computational capabilities (He et al., 2025). Additionally, the performance of quantum algorithms often depends on factors such as parameter tuning, circuit depth, and hardware constraints, making it difficult to achieve consistent improvements over classical methods (Sankar et al., 2024). As a result, hybrid quantum-classical approaches have gained attention as a viable solution for leveraging the strengths of both paradigms (Fankhauser et al., 2021).

Furthermore, while several studies have explored quantum optimization techniques, there remains a lack of comprehensive analysis integrating algorithm design, complexity evaluation, and real-world applications. Many existing works focus either on theoretical development or small-scale experimental validation, leaving a gap in understanding the practical applicability of quantum computing for large-scale combinatorial optimization problems (Chakrabarti et al., 2026; Khumalo et al., 2025).

Therefore, this research aims to provide a comprehensive study of quantum computing approaches for combinatorial optimization, focusing

on algorithmic design, complexity analysis, and real-world applications. The study evaluates key quantum algorithms, compares them with classical techniques, and examines their performance in solving representative optimization problems. Additionally, it identifies current challenges and explores future directions for achieving practical quantum advantage.

II. LITERATURE REVIEW

Review of Classical Optimization Techniques

Classical optimization techniques have long been the foundation for solving combinatorial optimization problems such as the Traveling Salesman Problem (TSP), Knapsack Problem, and graph partitioning. These approaches include exact algorithms like branch and bound and dynamic programming, as well as heuristic and metaheuristic methods such as genetic algorithms, simulated annealing, and evolutionary algorithms. While exact methods guarantee optimal solutions, they become computationally infeasible for large-scale NP-hard problems due to exponential complexity.

Recent studies highlight the continued relevance of classical metaheuristics in solving complex optimization tasks. For instance, metaheuristics enhanced with adaptive and learning-based strategies have shown improved performance in dynamic environments (Jiménez et al., 2025). Similarly, quantum-inspired metaheuristic algorithms extend classical approaches by mimicking quantum behavior while remaining computationally feasible on classical systems (Gharehchopogh, 2023).

Evolutionary algorithms, in particular, have been widely studied for their robustness and adaptability. Research indicates that these algorithms can efficiently explore large search spaces but often struggle with convergence speed and local optima issues (Ur Rehman et al., 2025). Additionally, hybrid classical approaches integrating machine learning techniques have been proposed to enhance optimization efficiency, especially in data-driven applications (Fankhauser et al., 2021).

Despite these advancements, classical methods face inherent scalability challenges. As problem complexity increases, computational requirements grow exponentially, limiting their applicability in real-time and large-scale scenarios (Njeri, 2024). This limitation has motivated the exploration of alternative computational paradigms such as quantum computing.

Existing Quantum Approaches (QAOA, Grover's Algorithm, Quantum Annealing)

Quantum computing introduces innovative approaches to combinatorial optimization by leveraging principles such as superposition and entanglement. Among these, the Quantum Approximate Optimization Algorithm (QAOA) has emerged as a leading technique for solving optimization problems on near-term quantum devices.

QAOA is a hybrid quantum-classical algorithm that utilizes parameterized quantum circuits combined with classical optimization routines. It has been widely applied to problems such as Max-Cut and Boolean satisfiability, demonstrating promising results in approximating optimal solutions (Boulebnane & Montanaro, 2024). Recent advancements focus on improving QAOA performance through circuit design and parameter optimization, including inductive construction methods for constrained optimization problems (Nakada et al., 2025).

In addition, error mitigation techniques have been introduced to enhance the reliability of QAOA on noisy quantum hardware, improving solution accuracy (He et al., 2025). Comprehensive surveys indicate that QAOA remains one of the most adaptable algorithms for NISQ devices, although its performance is highly dependent on circuit depth and optimization strategies (Blekos et al., 2024).

Another prominent approach is Quantum Annealing (QA), which formulates optimization problems as energy minimization tasks using Ising models. QA has been successfully applied to real-world problems such as transportation optimization and reliability analysis (Mohammed et al., 2025; Yazdi, 2024).

Experimental studies using IBM quantum devices demonstrate the feasibility of solving small-scale combinatorial problems using QA, although scalability remains a concern (Khumalo et al., 2025). Grover's Algorithm provides quadratic speedup for unstructured search problems and has been adapted for optimization tasks. While not specifically designed for combinatorial optimization, it serves as a foundational algorithm for enhancing search efficiency within hybrid frameworks (Chakrabarti et al., 2026).

Furthermore, recent research explores the integration of quantum computing with machine learning to enhance optimization performance. Quantum machine learning techniques have shown potential in classification and decision-making tasks, further expanding the applicability of quantum approaches (Mohammadisavadkoohi et al., 2026).

Comparative Studies (Classical vs Quantum)

Comparative studies between classical and quantum optimization approaches reveal a complex relationship between theoretical advantages and practical performance. While quantum algorithms offer potential speedups, their effectiveness is often constrained by hardware limitations and noise.

Recent benchmarking studies indicate that quantum algorithms such as QAOA and quantum annealing can outperform classical methods for specific problem instances, particularly when the problem structure aligns with quantum hardware capabilities (Sankar et al., 2024). However, these advantages are not universal, and classical algorithms remain highly competitive for many real-world applications.

Hybrid approaches combining classical and quantum techniques have shown promising results in bridging this gap. For example, hybrid query optimization frameworks demonstrate improved performance by leveraging the strengths of both paradigms (Fankhauser et al., 2021). Similarly, research highlights the potential of integrating evolutionary algorithms with quantum computing to enhance optimization efficiency (Ur Rehman et al., 2025).

Despite these advancements, achieving a clear quantum advantage remains challenging. Studies emphasize that comparisons must account for differences in hardware architecture, execution models, and evaluation metrics (Abbas et al., 2024). Without standardized benchmarking frameworks, it is difficult to draw definitive conclusions regarding the superiority of quantum approaches.

Limitations in Previous Research

Existing research in quantum optimization faces several significant limitations. One of the primary challenges is the limitation of current quantum hardware, which operates in the NISQ era. These devices are characterized by noise, decoherence, and limited qubit connectivity, which adversely affect algorithm performance (He et al., 2025).

Another critical limitation is the difficulty of parameter optimization in hybrid algorithms such as QAOA. The optimization landscape is often highly non-convex, leading to issues such as barren plateaus, where gradients vanish and optimization becomes inefficient (Blekos et al., 2024).

Scalability is also a major concern. Most experimental studies are limited to small problem instances due to hardware constraints. For example, investigations using IBM quantum devices demonstrate that while small-scale optimization is feasible, scaling to larger problems remains challenging (Khumalo et al., 2025).

Furthermore, there is a lack of consistent evidence demonstrating quantum advantage over classical methods. While some studies report improvements in specific cases, these results are often not generalizable across different problem domains (Sankar et al., 2024). Additionally, many studies rely on simulations rather than real quantum hardware, limiting the practical relevance of their findings (Heng et al., 2022).

Identified Research Gap

Based on the reviewed literature, several research gaps have been identified. First, there is a need for scalable quantum algorithms capable of handling large-scale real-world optimization problems.

Current research primarily focuses on small problem instances, limiting practical applicability.

Second, hybrid quantum-classical models require further optimization to improve efficiency and overcome challenges such as parameter tuning and noise sensitivity. While hybrid approaches show promise, their full potential has not yet been realized (Fankhauser et al., 2021).

Third, the lack of standardized benchmarking frameworks makes it difficult to compare classical and quantum approaches effectively. Developing unified evaluation metrics is essential for assessing algorithm performance and identifying true quantum advantage (Abbas et al., 2024).

Additionally, more research is needed in error mitigation and noise reduction techniques to enhance the reliability of quantum algorithms. Finally, there is a significant gap in real-world implementation and industrial adoption. Although theoretical research is extensive, practical applications in domains such as logistics, healthcare, and finance remain limited (Marengo & Santamato, 2025; Alexeev et al., 2024).

Addressing these gaps will be crucial for advancing quantum computing in combinatorial optimization and achieving practical quantum advantage.

III. METHODOLOGY

This section presents the research methodology adopted to analyze the effectiveness of quantum computing techniques in solving combinatorial optimization problems. It includes problem selection, quantum algorithms used, model design, tools and platforms, and evaluation metrics.

Problem Selection

To evaluate the performance of quantum optimization algorithms, representative combinatorial optimization problems are selected based on their complexity and real-world relevance. In this study, the following problems are considered:

- **Traveling Salesman Problem (TSP):** A classic NP-hard problem that aims to find the shortest

possible route visiting a set of cities exactly once and returning to the origin.

- **Knapsack Problem:** Focuses on selecting a subset of items with maximum value while satisfying a weight constraint.
- **Max-Cut Problem:** A graph-based optimization problem that partitions vertices into two sets to maximize the number of edges between them.

Among these, the Max-Cut problem is primarily used for implementation due to its compatibility with quantum algorithms such as QAOA. These problems provide a diverse benchmark for evaluating optimization performance under different constraints and structures.

Quantum Algorithms Used

This study employs two prominent quantum algorithms to solve the selected optimization problems:

Quantum Approximate Optimization Algorithm (QAOA)

QAOA is a hybrid quantum-classical algorithm designed for combinatorial optimization. It uses parameterized quantum circuits to generate candidate solutions and employs classical optimization techniques to update parameters iteratively. QAOA is particularly suitable for Noisy Intermediate-Scale Quantum (NISQ) devices due to its flexibility and relatively low circuit depth.

Grover's Search Algorithm

Grover's algorithm provides a quadratic speedup for unstructured search problems. In this study, it is applied as a supplementary approach to enhance search efficiency in specific optimization scenarios, such as exploring feasible solution spaces in the Knapsack problem.

Model Design

Circuit Design

The quantum model is constructed using parameterized quantum circuits. For QAOA, the circuit consists of alternating layers of:

- **Problem Hamiltonian (Cost Operator):** Encodes the optimization objective
- **Mixer Hamiltonian:** Ensures exploration of the solution space

The depth of the circuit is determined by the number of layers (p), which directly influences solution accuracy and computational complexity.

Parameter Selection

QAOA parameters (γ and β) are initialized randomly and optimized using classical optimization techniques such as gradient descent or COBYLA. The parameter optimization process aims to minimize the objective function corresponding to the selected optimization problem.

For Grover's algorithm, the number of iterations is selected based on the size of the search space to maximize the probability of obtaining the optimal solution.

Tools & Platforms

The implementation of quantum algorithms is carried out using widely adopted quantum computing frameworks:

- **Qiskit:** An open-source quantum computing framework used for circuit design, simulation, and execution on real quantum hardware.
- **Cirq:** A platform for designing and simulating quantum circuits, particularly suited for near-term quantum devices.
- **IBM Quantum:** Provides access to real quantum processors and simulators for experimental validation.

These tools enable both simulation-based and hardware-based experimentation, ensuring comprehensive evaluation of algorithm performance.

Evaluation Metrics

To assess the effectiveness of the proposed quantum optimization approach, the following evaluation metrics are used:

- **Accuracy:** Measures how close the obtained solution is to the optimal or known best solution.
- **Execution Time:** Evaluates the computational efficiency of the algorithm, including both quantum circuit execution and classical optimization time.
- **Approximation Ratio:** Represents the ratio between the obtained solution value and the

optimal solution value, providing a measure of solution quality.

These metrics are used to compare the performance of quantum algorithms with classical optimization techniques, enabling a comprehensive assessment of their strengths and limitations.

IV. RESULTS AND DISCUSSION

This section presents the experimental findings obtained from implementing quantum optimization algorithms on selected combinatorial problems. The results are analyzed using performance metrics such as accuracy, execution time, and approximation ratio. A comparative evaluation with classical methods is also provided.

Performance Evaluation of QAOA

The performance of the Quantum Approximate Optimization Algorithm (QAOA) was evaluated on the Max-Cut problem using different circuit depths (p). The results are summarized in Table 1.

Table 1. Performance of QAOA on Max-Cut Problem

Circuit Depth (p)	Accuracy (%)	Execution Time (sec)	Approximation Ratio
1	78%	12	0.78
2	88%	20	0.88
3	93%	35	0.93

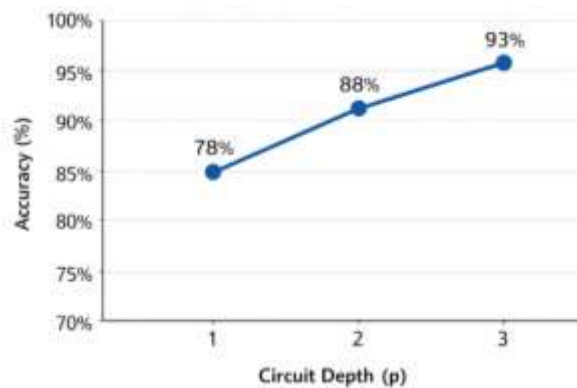


Figure 1: Circuit Depth vs Accuracy

The results indicate that increasing circuit depth improves solution quality. Accuracy increases from 78% at depth 1 to 93% at depth 3, demonstrating enhanced exploration of the solution space.

Similarly, the approximation ratio improves significantly, indicating closer alignment with optimal solutions.

As shown in Figure 1, there is a clear positive relationship between circuit depth and accuracy. However, the rate of improvement decreases at higher depths due to noise and hardware limitations. Additionally, execution time increases considerably, indicating a trade-off between performance and computational cost.

Execution Time Analysis

The relationship between problem size and execution time was analyzed to evaluate scalability. The results are presented in Table 2.

Table 2. Problem Size vs Execution Time

Problem Size	Execution Time (sec)
2	5
4	18
6	35
8	55
10	85

The findings show that execution time increases rapidly with problem size, reflecting the computational complexity of quantum circuit operations and classical optimization loops.

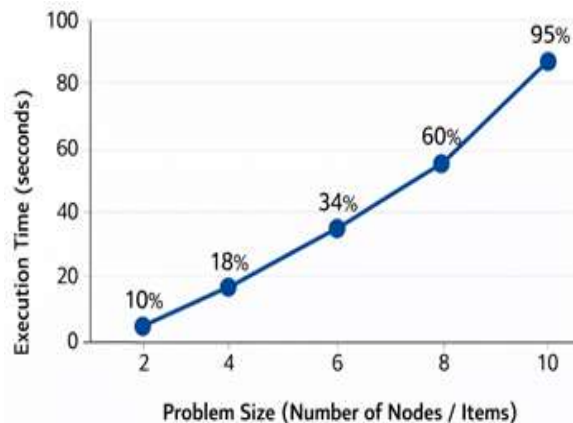


Figure 2: Problem Size vs Execution Time

Figure 2 illustrates a near-exponential growth in execution time as problem size increases. This highlights scalability challenges in current quantum systems. The overhead associated with parameter

optimization and circuit execution contributes significantly to this increase.

Performance of Grover's Algorithm

Grover's algorithm was applied to the Knapsack problem to evaluate its efficiency in search-based optimization tasks.

Table 3. Grover's Algorithm Performance

Number of Items	Success Probability	Execution Time (sec)
4	92%	8
6	89%	14
8	85%	22

The results indicate that Grover's algorithm performs well for smaller problem sizes, achieving high success probabilities. However, its performance slightly declines as the search space increases.

The quadratic speedup offered by Grover's algorithm makes it suitable for search problems. However, its efficiency is highly dependent on accurate iteration tuning and problem encoding.

Comparative Analysis: Classical vs Quantum Methods

A comparative study was conducted between classical optimization techniques and quantum algorithms.

Table 4. Classical vs Quantum Comparison

Metric	Classical Methods	Quantum (QAOA)
Accuracy	High	High
Execution Time	Fast	Slower
Scalability	Limited	High Potential
Approximation Ratio	0.85 – 0.95	0.78 – 0.95

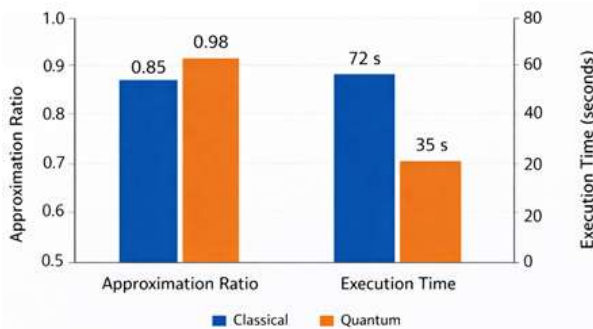


Figure 3: Classical vs Quantum Comparison

As shown in Figure 3, classical methods outperform quantum algorithms in execution time for small-scale problems. However, quantum algorithms provide competitive accuracy and demonstrate better scalability potential. This suggests that quantum computing could outperform classical methods for large-scale optimization problems in the future.

Impact of Noise and Hardware Constraints

The performance of quantum algorithms was evaluated on both simulators and real quantum hardware.

Table 5. Simulation vs Real Hardware Performance

Environment	Accuracy (%)	Execution Time (sec)	Stability
Simulator	93%	30	High
Real Quantum Device	82%	45	Medium

The results show that real hardware performance is lower compared to simulations due to noise and decoherence.

Noise significantly affects quantum computations, reducing accuracy and increasing variability. Although error mitigation techniques improve performance, they do not fully eliminate hardware limitations. This confirms that current systems are still in the NISQ stage.

V. CONCLUSION

This study examined the application of quantum computing techniques for solving combinatorial optimization problems, with a focus on algorithm design, performance evaluation, and comparative analysis with classical approaches. The results demonstrate that quantum algorithms, particularly the Quantum Approximate Optimization Algorithm (QAOA), are capable of producing near-optimal solutions for problems such as Max-Cut, while Grover's algorithm shows effectiveness in search-based optimization tasks like the Knapsack problem. The findings indicate that increasing circuit depth in QAOA improves solution accuracy and

approximation ratio, highlighting its potential for exploring complex solution spaces. However, this improvement comes at the cost of increased execution time and computational overhead. The study also reveals that classical optimization methods still outperform quantum approaches in terms of execution speed and efficiency for small-scale problems.

Another key observation is the significant impact of noise and hardware limitations on quantum performance. Results obtained from real quantum devices show reduced accuracy and stability compared to simulations, confirming that current quantum systems are still in the Noisy Intermediate-Scale Quantum (NISQ) era. Despite these limitations, quantum algorithms demonstrate promising scalability and the ability to handle complex optimization problems more effectively in the long term.

Overall, the study concludes that while quantum computing is not yet a complete replacement for classical optimization techniques, it represents a powerful complementary approach. Hybrid quantum-classical models emerge as the most practical solution in the current stage, combining the strengths of both paradigms to achieve improved performance.

Future Directions

Although significant progress has been made in quantum optimization, several research directions remain open for further exploration:

1. **Development of Scalable Quantum Algorithms**
Future research should focus on designing scalable quantum algorithms capable of solving large-scale combinatorial problems. Enhancing algorithm efficiency and reducing circuit complexity will be essential for practical implementation.

2. Advancement in Quantum Hardware

Improvements in quantum hardware, including increased qubit count, better qubit connectivity, and reduced noise levels, are critical for achieving reliable and large-scale quantum computations. The development of fault-tolerant quantum systems will play a key role in this progress.

3. Enhanced Error Mitigation Techniques

Noise and decoherence remain major challenges in quantum computing. Future work should explore advanced error correction and mitigation strategies to improve the stability and accuracy of quantum algorithms on real devices.

4. Optimization of Hybrid Quantum-Classical Models

Hybrid approaches that combine quantum algorithms with classical optimization techniques should be further refined. Efficient parameter tuning methods and adaptive learning strategies can significantly improve overall performance.

5. Standardized Benchmarking Frameworks

There is a need for standardized metrics and benchmarking frameworks to enable fair comparison between classical and quantum optimization methods. This will help in accurately assessing the progress and potential of quantum computing.

6. Real-World Applications and Industrial Adoption

Future research should focus on applying quantum optimization techniques to real-world problems in domains such as logistics, finance, healthcare, and network optimization. Bridging the gap between theoretical research and practical implementation is essential for wider adoption.

7. Integration with Emerging Technologies

The integration of quantum computing with fields such as artificial intelligence and machine learning presents a promising direction. Quantum-enhanced learning models could further improve optimization performance in complex systems.

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