

Application of Mathematics in Modern Science and Technology

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Abstract- Mathematics plays a fundamental role in the development of modern science and technology. Many real-world problems arising in physics, biology, engineering, environmental science, computer science, and medical research can be analysed and understood through mathematical modelling. Mathematical models simplify complex systems and express them in the form of equations, allowing scientists and researchers to study the behaviour of changing quantities with accuracy and clarity. Among the various mathematical tools, differential equations occupy significant place because they describe relationships involving rates of change and evolving systems. Differential equations are widely used to model natural phenomena such as population growth, radioactive decay, heat transfer, and spread of diseases, motion of falling bodies, electrical circuits, and chemical reactions. In particular, first-order differential equations are highly useful in understanding growth and decay processes where the rate of change of a quantity is proportional to the quantity itself. These equations help predict future behaviour based on present conditions and are therefore valuable in scientific planning and technological applications. This paper discusses the applications of mathematics in modern science and technology with special emphasis on differential equations and population growth models. The law of natural growth is introduced by assuming that the rate of increase of a population is directly proportional to the existing population. By forming and solving the corresponding first-order differential equation, an exponential growth model is obtained. The paper also presents a real-life application involving bacterial population growth to demonstrate how mathematical techniques are used for prediction and analysis. The study highlights how mathematics supports scientific discoveries, technological innovations, and decision-making processes in modern society. From predicting epidemics such as Covid-19 to analysing ecological systems and resource planning, mathematical methods provide reliable and efficient solutions. Thus, mathematics serves as the backbone of modern science and technology and continues to contribute significantly to future advancements in research and innovation.

Keywords: Mathematics, Differential Equations, Population Growth, Mathematical Modelling, Science and Technology, Exponential Growth, Applications of Mathematics, First-Order Differential Equations.

I. INTRODUCTION

Mathematics helps in converting real-life problems into mathematical models. These models simplify complex problems and represent them in the form of mathematical equations. Differential equations play a prominent role in mathematical modeling. These models involve ordinary and partial differential equations that relate functions representing rates of change to their derivatives.

Therefore, differential equations are particularly useful for understanding how quantities evolve over time. First order differential equations are applied in various fields such as growth and decay processes, mixing problems, Newton's law of cooling, motion of falling bodies, and electrical circuits. Growth and decay applications of differential equations are important for modeling quantities that change proportionally over time.

Assume that the population grows at a rate directly proportional to the population present at that time.

II. MATHEMATICAL MODELING

These models involve ordinary and partial differential equations that relate a function representing rates of change to its derivatives. Therefore they are particularly suitable for modeling mathematical equations to understand how things evolve. In my particular observation during the COVID-19 period, the number of infection cases increased rapidly with time. And also during major sports events such as world cricket finals, the number of viewers may increase rapidly with time. This situation can be modeled by the differential equation.

Advantages of Mathematical Modeling:

- Predicts future population
- Helps government planning
- Useful in environmental studies
- Assists medical research.

Formation of Differential Equations:

In this model, two variables are assumed to be directly proportional to each other, leading to a first-order first-degree differential equation.

$$\frac{dP}{dt} \propto P$$

$$\frac{dP}{dt} = kP$$

1. **Cricket match Viewers:** $\frac{dv}{dt} = kv$

Where v represents the number of viewers at time for cricket viewers

2. **Spread of Infections:** $\frac{dI}{dt} =$

Where I represents infected people

3. **Population Growth:** $\frac{dP}{dt} = kP$

Where P is human population increases with time

These applications can be observed directly in our Human life

Law of natural growth

Let P(t) be the population at any time t. Assume that population grows at a rate directly proportional to the amount of population present at that time then the differential equation is the first order, First degree linear equation.

$$\frac{dP}{dt} = kP$$

Where P = Population

t = time

k = growth constant

Separating variables

$$\frac{dP}{P} = k dt$$

Integrating both side

$$\int \frac{1}{P} dP = \int k dt$$

$$\ln P = kt + C$$

$$P = P_0 e^{kt}$$

Where P_0 = Initial population

e = exponential constant

- If $K > 0$ population increases
- If $K < 0$ population decreases

III. GROWTH AND DECAY MODEL

Growth and decay models describe situations in which the rate of change of a quantity depends on the amount present at that time. If the rate of increase of a quantity is directly proportional to the existing quantity, then the process is called growth. Similarly, if the rate of decrease is proportional to the existing quantity, then the process is called decay. These models are commonly represented by first-order differential equations.

Mathematically, the model is written as

$$\frac{dP}{dt} = kP$$

Where P represents the quantity at time t . and k is a proportionally, and if $K > 0$ the quantity grows exponentially, and if $K < 0$ the quantity decreases exponentially.

Growth and decay models are widely used in population studies, radioactive decay, and spread of diseases, bacteria growth, financial analysis, and environmental science. These models help scientists and researchers predict future behaviour and understand changing systems more effectively.

Solved Problem:

A bacterial population B is known to have a rate of growth Propanol to B itself. If between noon and 2pm the population triples. At what time, no controls being exerted, should B become 100 times what it was at noon?

General equation of the particular present

$$\frac{dB}{dt} = kB$$

Whose solution is $B(t) = ce^{kt}$

Let B_0 be the initial population at $t=0$ using this condition

$$B_0 = ce^0 \quad \text{i.e. } B_0 = c$$

Then $B = B_0 e^{kt}$

Since population triples i.e. becomes $3B_0$ between noon and 2 pm i.e. in two hours, we use this condition to find k

$$3B_0 = B_0 e^{2k}$$

$$K = \frac{1}{2} \ln 3 = 0.54930$$

Hence the population rule is

$$B(t) = B_0 e^{0.54930t}$$

Now to find the time at which population becomes 100 times the original i.e. $100B_0$

We put $B = 100B_0$ in the above equation and solve for t

$$100B_0 = B_0 e^{0.54930t}$$

$$\text{Solving } t = \frac{\ln 100}{0.54930} = 8.3837015$$

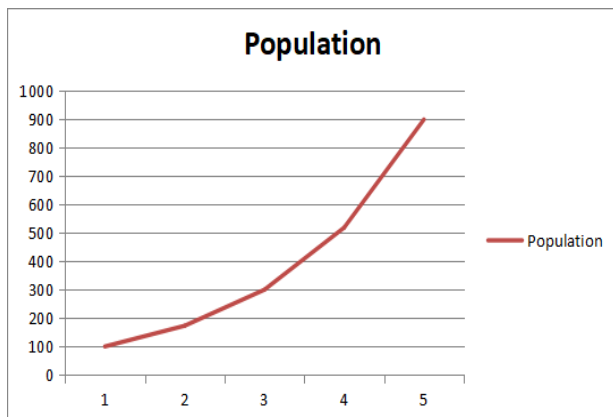
At 8.38370 pm population becomes 100 times the original population.

Convert the decimal hours to minutes:

$$0.3836 \times 60 \text{ minutes} = 23 \text{ minutes}$$

Adding 8 hours and 23 minutes to noon gives 8:23 pm

$K > 0$ graphs

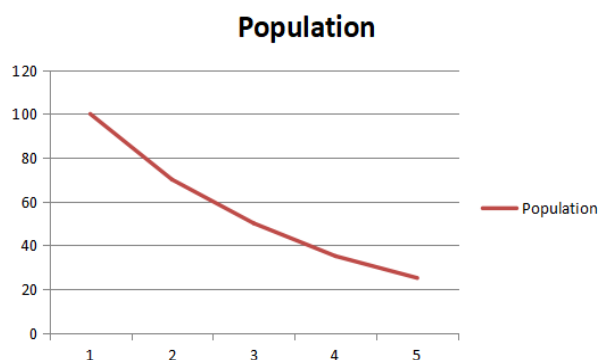


- If $K > 0$, the quantity increases exponentially with time and represents a growth process.
- Upward curve
- Faster growth later
- Curve never touches time axis.
- Growth becomes faster with time
- Larger population gives larger growth rate
- The graph shows exponential population growth.
- Initially the population increases slowly. But after some time it grows rapidly because the growth rate depends on existing population.

Examples

- Human population increases with time
- Bacteria multiply rapidly
- Infected people.
- During final overs,
- Online viewers increase rapidly.
- Viral videos
- Trending news

$K < 0$ graph



When $K < 0$ the quantity decreases exponentially with time.

Initially the decrease is rapid, but later it becomes slower.

The curve approaches the time axis but never touches it.

- Population decreases with time
- Decay becomes slower gradually
- Curve approaches the axis asymptotically
- Used in radioactive decay and medicine

Examples for $k < 0$

- Radioactive decay
- Cooling process
- Decrease of medicines in blood
- Decline of virus cases after controls

Real-Life Application:

- Human population studies
- Bacteria growth
- Spread of epidemics
- Ecology
- Resource planning

IV. CONCLUSION

If there is a proportional relationship between two variables, then a first order, first degree differential equations can be formed between them. Mathematical methods simplify complex real-life problems and make them easier to understand. Differential equation helps in predicting population changes and support scientific planning. They also play an important role in modelling real-world problems such as population growth. Mathematics helps scientists make predictions and better decisions for society.

In modern science and technology, mathematical modelling has become an essential tool for analysing real-life phenomena. Differential equations not only describe growth and decay processes but also help

researchers understand changing systems in biology, engineering, environmental science, and communication technology. Thus, mathematics continues to contribute significantly to scientific progress and technological innovation.

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