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Analysis of Finite Capacity Queueing Model with No-Passing, Reverse Balking and Reneging

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Abstract- In this study, we present Markovian queueing model with finite capacity, reverse balking and reneging of impatient customers. Some performance measures including queue length, waiting time, transient probabilities, some special case and optimal solution are obtained by applying numerical techniques. To verify the numerical results, computational software has been used.

Keywords: Queueing Model; No-passing; Reverse Balking and Reneging.

I. INTRODUCTION

In a present scenario, queueing model plays a very important role. Queueing model is an arrangement of arrival rate as well as service rate. It is an initial requirement of every business, bank, industry, factory, restaurant, multinational company, online marketing, hospitals, finance filed, tele communication. Queueing models gives an idea with the help of which we can improve our service and deal with more customers at a time. You can reduce waiting time and get more profit with minimum cost. Queueing models are very effective in congestion situation. In congestion condition, customers often leave the system due to which we have to face huge losses. Often the customers leave after seeing too much crowd in the system, without even entering the system (i.e. balking). After entering the system, if the waiting time is getting longer than the customer leaves from there (i.e. reneging).

In this paper we talk about some problems related to finance, investment, Mutual fund company and banks. Investment related business, customer would prefer to go to the place where the customers are already available or where the number of customers is more. In mutual funds, the customers want to invest where the other customers have already invested. Here the customer believes that he will make profit here because the other customers has also made his/ her investment here.

II. LITERATURE REVIEW

A key idea in queuing theory is the queueing model without passing, sometimes known as a single-server queue or a basic queuing system. In this model, clients show up at a service location and wait in line to speak with a single server. After being served, they are removed from the system. Customers are served in the order that they arrive; there is no system in place enabling them to pass one another in the queue.

The research on no-passing in queuing systems can be abridged as follows. Washburn, A. (1974) studied queuing model with no-passing and the focus is likely on analyzing a specific type of queueing system where multiple servers are employed, but customers are not allowed to pass each other in the queue. Rego, V. J., & Ni, L. M. (1985) suggested token-passing network designers and engineers can better understand how their networks behave under various circumstances and adjust performance by using queuing models.

They can aid in the detection of possible bottlenecks, forecast the effects of modifications to the network configuration or protocol, and direct the creation of communication systems that are more dependable and efficient. Gelenbe, E. (1990) proposed insightful information on the Connection Machine's performance traits and capabilities that advances knowledge about parallel computing systems and their uses. For researchers, engineers, and practitioners interested in high-performance computing and parallel processing, it probably acts as a major reference.

Jain.M.et al (1996) suggested Queueing theory is highly recommended for experts in network processes, computer networks, and technology who often operate multi-server systems with priority requirements. Jain and Singh (2003) assisted in the study of queueing systems by investigating a Markovian queueing model that contains waiting time, a loss, pass-through discipline, and removable excess servers. Their work provides vital insights into predicting system performance and understanding the operating features of such service systems. The findings offer suggestions for modifying system settings and enhancing efficiency in real applications.

Breuer and Baum (2005) gave a comprehensive introduction to matrix-analytic techniques and queueing theory, highlighting their value in the study of intricate stochastic systems. Their work serves as a significant resource for academics, practitioners, and students by offering the theoretical underpinnings and analytical tools necessary to analyze, simulate, and increase the efficiency of queueing systems across multiple application domains.

By presenting and analyzing queueing models containing balking and reneging behaviors, Ancker Jr. and Gafarian (1963) made a substantial contribution. Their study improved our understanding of customer-driven phenomena in queueing system design and performance evaluation and set the stage for further research in this field.

Ancker Jr, C. J. &Gafarian, A. V. (1963 proposed Several Queuing Issues with Balking and Reneging—II," which delves further into the subject and examines several facets of traditional queuing models with balking and reneging.

Abou-El-Ata, et al (1992) In this paper, work has been done on the general bulk function, and the steady-state distribution for a single unit has been derived. Along with this, the number of units present in the system and in the queue has also been analyzed. A model has been defined that promises good service, and some special cases have been discussed as well. These cases explain how to manage time effectively and how to arrange the units efficiently. Liao, P. Y. (2011) this paper has been created to estimate business loss. In this model, the balking index and reneging set have been used. With this, we can increase customer satisfaction and take the business to the next level by using different management technologies. Kumar, R. (2013) proposed model, a multi-server queuing model has been used, where the focus is basically on reneging customers. The profit part of this model is totally based on reneging customers. The more reneging customers there are, the more service we will provide, which will result in higher profit for them.

Wang, Q., & Zhang, B. (2018 This model illustrates reverse reneging and reverse walking behavior. It explains that if the queue length becomes too long, customers will not join the queue and will not be satisfied with the company. As a result, it becomes unlikely that the customer will reconnect with the company in the long term, which can cause heavy losses for the company. To prevent these heavy losses, we have developed a reverse reneging and reverse walking model so that the company can gain significant benefit. Kuaban, G. S. et al (2020) defined a healthcare model that focuses on healthcare management, ensuring that if a patient is in line, they can receive their service or treatment within the given time and it does not get missed.

If it does get missed, there is a guarantee that the next customer or patient behind them will also be served meaning they can also leave the queue. If this doesn't happen, the healthcare system could face heavy losses. To prevent this, a management model has been defined in which queue length control, reverse reneging, and balking behavior are clearly defined. Time management is also taken into account, so that the healthcare system gets more patients and can deal with as many patients as possible. Kumar, R., & Som, B. K. (2014) suggested a model, this model is built as an investor model. In it, the larger the system size, the lower the chances of customers walking away. This model is very important and beneficial for investors, as it helps them get more customers who invest in their business, leading to higher profits. For this, parameters like performance and waiting time have been defined and calculated, and a numerical formula has also been provided here.

Awasthi (2018) proposed a model in which, address the decline in service quality in the industry and service sector, which is reducing customer trust in companies. To resolve these customer issues, a finite capacity reverse reneging model has been defined. In this model, customers are assured that they will receive service on time. This assurance makes customers willing to wait even in long queues, which provides significant benefit to the company and allows it to serve more customers effectively. Som and Seth (2019) proposed a model, the customer is encouraged to stay connected with our company by providing good service.

A model has been prepared where deals and discounts are discussed. The idea is that if the deal is good and the discount is attractive, the customer will stay with the company for a longer time. Keeping the long-term goal in mind, the aim is to create a model that benefits the company, attracts more customers, and encourages them to come back and use the service again.

Kumar and Som (2020) in this model, a multi-server queue system has been developed. The main purpose is to ensure that as many customers as possible connect with this model or with the company, so that they get maximum benefit. To attract more customers, the seating arrangement has been increased, which is why we call it a multi-server system. In this, the concept of reverse reneging and balking has been used. Som et al. (2020) In this model, the customer is encouraged to stay connected with our company by providing good service. A model has been prepared where deals and discounts are discussed.

The idea is that if the deal is good and the discount is attractive, the customer will stay with the company for a longer time.Keeping the long-term goal in mind, the aim is to create a model that benefits the company, attracts more customers, and encourages them to come back and use the service again.

Zaghouani et al. (2022) In this study, the focus is on customer behavior. The study is entirely based on radio networks, in radio networks, customers usually prefer to go to places where there are more customers. However, due to the high number of customers, there are also situations where a customer chooses to leave the network and switch to another one. Keeping this in mind, the authors have developed a model that defines and analyzes two behaviors: reverse balking and reneging. Rajeswari and Ritha (2022) investigated a multi-server queueing model operating in an imprecise environment, incorporating customer impatience and reverse balking behavior. Their study enhanced the understanding of how such customer behavior influence system dynamics under uncertainty. Using a range of analytical techniques, they evaluated key performance characteristics of the model and provided insights into its effectiveness in representing real-world service systems.

III. MODEL DESCRIPTION

In this paper, we discussed finite capacity (L) queueing model with no-passing, reverse balking and reneging. Arrival of customer consider as Poisson process with mean rate $(L-\boldsymbol{\alpha}\beta)]\ \varrho)$. Customers are served

$(\boldsymbol{\alpha}+\beta)$

by service rate σ with exponential distribution and reneging times also exponentially distributed with parameter δ.

We considered some cases as follows –

- There is no customer in the system (i.e. system is empty).
- There is one customer in the system.

If there is no customer in the system then the probability of the customer not joining the system will be q′ and probability of the customer joining the system will be p′ = (1 − q′).

If there is one customer in the system then the probability of the customer not joining the system will be $\left(\frac{m}{(1-n)} - 1\right)$ and

(L−1) probability of the customer joining the system will be $\left(\frac{m}{(L-1)}\right)$

IV. MATHEMATICAL ANALYSIS

In this section, we analysed the mathematical calculation for different performance.

Some mathematical notations are as follows-pm - The probability that there are m customers in the system.

α, β - parameters.

E[L] - Average number of customers in system.

W - Expected waiting time of customers in a system.

Rr - Average reverse reneging rate.

Br - Average reverse balking rate.

- Arrival rate – arrival rate of customer is defined as-

$$\varrho_m = \begin{cases} \left(\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]\varrho\right) & m < L \\ 0 & m \geq L \end{cases} \quad (1)$$

- Service rate – service rate of customer is define as-

$$\sigma_m = \begin{cases} \sigma & m \leq L \\ 0 & m > L \end{cases} \quad (2)$$

To find the steady state probability, we are using rate-equality principal (i.e. rate up = rate down).

$$\varrho m\ pm = \sigma m\ pm+1 \ (3)$$

from equation (1) and (2) in equation (3), we get

$$\left(\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]\varrho\right)p_m = \sigma \ \ p_{m+1}$$

$$p_{m+1} = \frac{1}{\sigma}\left(\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]\varrho\right)p_m$$

$$p_{m+1} = \left(\frac{1}{\sigma}\left(\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]\varrho\right)\right)^2 p_{m-1}$$

$$p_{m+1} = \prod_{i=0}^{m} p_0 p'\left(\frac{1}{\sigma}\right)^i \left(\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]\varrho\right)^i$$

$$p_m = \prod_{i=0}^{m-1} p_0 p'\left(\frac{1}{\sigma}\left(\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]\varrho\right)\right)^i \quad (4)$$

Where m = 1,2,3,4...........L,

V. NORMALISED CONDITION

$$\sum_{m=0}^{\infty} p_m = 1$$

We get,

$$p_0 = \frac{1}{1+\sum_{m=0}^{\infty}\prod_{i=0}^{m-1} p'\left(\frac{1}{\sigma}\left(\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]\varrho\right)\right)^i} \quad (5)$$

For steady state series should be convergent.

VI. SOME PERFORMANCE MEASURES

Some performance measures are average number of customers, average rate of reverse reneging, average rate of reverse balking and expected waiting rime.

- **Average number of customers.** L

$$E[L] = \sum_{m=0}^{L} m p_m$$

$$E[L] = \sum_{m=0}^{L} m\left[\prod_{i=0}^{m-1} p_0 p'\left(\frac{1}{\sigma}\left(\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]\varrho\right)\right)^i\right] (6)$$

- **Average rate of Reverse Reneging.**

$$R_r = \sum_{m=1}^{L}[L-(m-1)]\delta p_m$$

$$R_r = \sum_{m=1}^{L}[L-(m-1)]\delta\prod_{i=0}^{m-1} p_0\left(\frac{1}{\sigma}\left(\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]\varrho\right)\right)^i (7)$$

- **Average rate of Reverse Balking.**

$$R_b = q'\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]p_0 + \sum_{m=1}^{L-1}\left[1 - \frac{m}{(L-1)}\right]\left(\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right)\varrho p_m$$

$$R_b = q\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]\varrho p_0 + \sum_{m=1}^{L-1}\left[1 - \frac{m}{(L-1)}\right]\left(\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right)\varrho\prod_{i=0}^{m-1} p_0\left(\frac{1}{\sigma}\left(\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]\varrho\right)\right)^i \quad (8)$$

- **Expected waiting time.**

$$W = \frac{E[L]}{\left(\left[\frac{(L-\alpha\beta)}{(\alpha+\beta)}\right]\varrho\right)} \quad (9)$$

VII. NUMERICAL ILLUSTRATION AND SENSITIVITY ANALYSIS

We can see from Table 1, figure 1 that the fixed value of p, = 0.2 ,average number of customers in system increases with increase in α (i. e. 1.5,1.6,1.7,1.8) as well as decrease in β (i.e. 1.9, 1.8, 1.7, 1.6, 1.5, 1.4, 1.3) in arrival rate. From Table 2, figure 2 that the fixed value of p, = 0.4 average number of customers in system increases with increase in α (i. e. 1.5,1.6,1.7,1.8) as well as decrease in β (i.e. 1.9, 1.8, 1.7, 1.6, 1.5, 1.4) in arrival rate. From Table 3, figure 3 that the fixed value of p, = 0.2 if we increase in α (i. e. 2, 2.1, 2.2, 2.3) as well as decreases in β (i.e. 1.7, 1.6, 1.5, 1.4, 1.3, 1.2, 1.1, 1) in arrival rate then average number of customers in system increases. From Table 4, figure 4, reverse reneging rate of customer increases with increase in arrival rate (i.e. 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6), and service rate (i.e. 6, 6.1, 6.2, 6.3). From Table 5, figure 5, average number of customers in system increases with increase in arrival rate (i.e. 1, 1.1, 1.2, 1.3) and service rate (i.e. 6, 6.1, 6.2, 6.3). From Table 6, figure 6, shows that the fixed values of p,= 0.4 and δ = 0.5, reverse reneging rate decreases with increase in α (i. e. 2, 2.5, 3, 3.5, 4) as well as increase in β(i.e. 2, 2.1, 2.2, 2.3 ,2.4).

Table 1- Average number of customers in system with $\mathbf{p}$, = $\mathbf{0.2}$

β/α	1.5	1.6	1.7	1.8
1.9	20.2513	9.7382	4.2691	1.4345
1.8	38.5593	19.4792	9.4611	4.1947
1.7	72.487	37.6317	19.2274	9.4611
1.6	135.8929	71.6462	37.6317	19.4792
1.5	255.6344	135.8929	72.487	38.5593
1.4	484.5622	258.4722	138.9985	75.0628
1.3	920.4237	495.1664	267.1594	145.4036

Table 2- Average number of customers in system with $\mathbf{p}$, = $\mathbf{0.4}$

β/α	1.5	1.6	1.7	1.8
1.9	40.5027	19.4765	8.5383	2.869
1.8	77.1186	38.9584	18.9222	8.3893
1.7	144.974	75.2634	38.4548	18.9222
1.6	271.7858	143.2924	75.2634	38.9584
1.5	511.2688	271.7858	144.974	77.1186
1.4	969.1243	516.9444	277.997	150.1255

Table 3- Average number of customers in system with $\mathbf{p}$, = $\mathbf{0.2}$

β/α	2	2.1	2.2	2.3
1.5	10.3115	4.8969	1.9476	0.348
1.4	21.5954	11.2208	5.4958	2.3331
1.3	43.5041	23.6035	12.5311	6.3394
1.2	86.2059	47.8527	26.417	14.3394
1.1	169.9893	95.5149	53.8251	30.2422
1	335.8571	189.8027	108.1165	61.8573

Table 4- Reverse Reneging rate with $\boldsymbol{\alpha} = \mathbf{2}$, $\boldsymbol{\beta} = \mathbf{2.5}$

ϱ/σ	6	6.1	6.2	6.3
1	3.1505	3.0861	3.0245	2.9654
1.1	3.5594	3.4835	3.411	3.3416
1.2	3.9998	3.9104	3.8252	3.744
1.3	4.4775	4.3723	4.2722	4.177
1.4	4.9998	4.8756	4.7579	4.6463
1.5	5.5757	5.4285	5.2897	5.1584
1.6	6.2169	6.0416	5.8769	5.7219

Table 5- Average number of customers in system with different arrival and service rate

ϱ/σ	6	6.1	6.2	6.3
1	1.2252	1.1592	1.1009	1.0491
1.1	1.8225	1.6809	1.5613	1.459
1.2	3.2398	2.8272	2.5105	2.2601
1.3	10.4127	7.1916	5.4987	4.4558

Table 6- Reverse reneging rate with $\mathbf{p}$,= $\mathbf{0.4}$,$\boldsymbol{\delta} = \mathbf{0.5}$

β/α	2	2.1	2.2	2.3
2	21.6059	17.2126	14.4363	12.4795
2.5	9.8441	8.7958	7.9339	7.2092
3	6.3542	5.7938	5.3026	4.8671
3.5	4.5231	4.1364	3.7869	3.4688
4	3.3513	3.0478	2.7683	2.5094

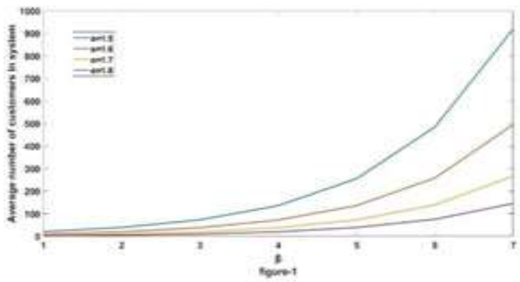

Figure 1- Average number of customers in system with p, = 0.2

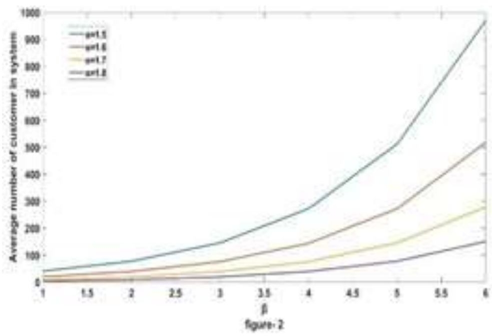

Figure 2- Average number of customers in system with p, = 0.4

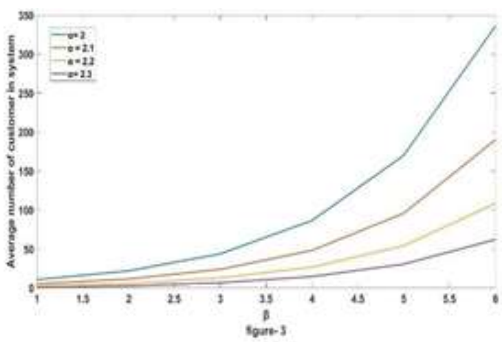

Figure 3- Average number of customers in system with p, = 0.2

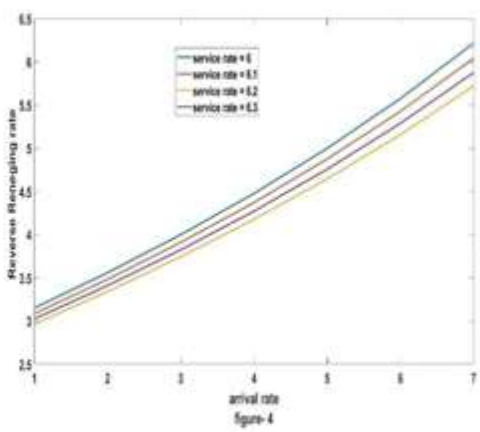

Figure 4- Reverse Reneging rate with α = 2, β = 2.5

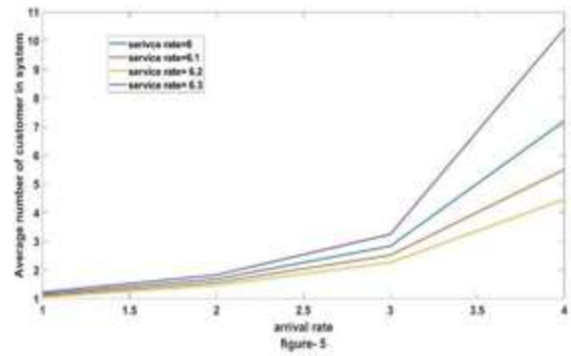

Figure 5- Average number of customers in system with different arrival and service rate

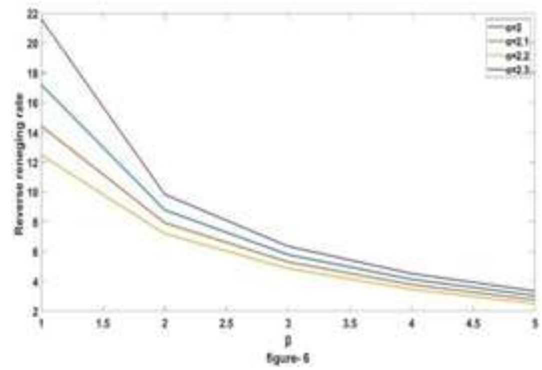

Figure 6- Reverse reneging rate with p, = 0.4, δ = 0.5

VIII. CONCLUSION

In the present study, a finite-capacity queueing model incorporating no-passing discipline, reverse balking, and reneging behavior has been developed and analyzed. The transient-state probabilities of the system have been derived to describe the time-dependent behavior of customers and service facilities under varying operating conditions. Unlike traditional queueing models, the proposed framework accounts for customers who may initially decide not to join the queue but later enter the system (reverse balking), as well as customers who may leave and subsequently rejoin the queue (reverse reneging).

These behavioral characteristics make the model more suitable for representing practical service environments where customer decisions are influenced by waiting conditions and system congestion. Using MATLAB software, we calculated the average number of customers in the system for different arrival rates, service rates, and reverse reneging rates.

To obtain the numerical results, we used MATLAB software. With its help, we have shown graphical representations and tables. In this paper future work can be expected to find the optimality. Bulk input and infinite capacity can also be study.

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