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Application of the Discrete Vortex Method to the Calculation of Aerodynamic Characteristics of Coaxial Helicopter Rotors

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Abstract- This paper presents the application of the discrete vortex method to calculating the aerodynamic characteristics of coaxial helicopter rotors. The study focuses on modeling the aerodynamic interaction between the upper and lower rotors, which rotate in opposite directions and create a complex unsteady flow field. Based on the discrete vortex method, the rotor blades are replaced by bound and free vortex segments, allowing the determination of pressure distribution, aerodynamic forces, moments, and thrust coefficients. A computational program was developed to evaluate the influence of initial azimuth angle and horizontal hinge distance on the aerodynamic performance of coaxial rotors. The obtained results provide a scientific basis for selecting geometric and kinematic parameters, supporting aerodynamic analysis, rotor design, and digital transformation in aviation education and research.

Keywords- Discrete vortex method, Coaxial rotors, Aerodynamic characteristics, Vortex wake.

I. INTRODUCTION

Coaxial rotor helicopters are one of the helicopter configurations with many outstanding advantages, such as compact overall dimensions, high lifting capability, and the ability to directly balance the reaction torque on the rotor shaft. Due to these characteristics, coaxial helicopters can achieve good flight stability, high maneuverability, and are suitable for operation in confined areas, complex terrain, and on naval vessels. However, the aerodynamic characteristics of coaxial rotors are more complicated than those of conventional single-rotor helicopters because the upper and lower rotors rotate in opposite directions and strongly interact with each other during operation.

The interaction between the two rotor systems creates a complex unsteady flow field, directly affecting the thrust coefficient, aerodynamic forces, moments, and flight stability of the helicopter. Therefore, the calculation and analysis of aerodynamic characteristics play an important role in the design, evaluation, and selection of suitable geometric and kinematic parameters for coaxial rotors. In particular, parameters such as the initial azimuth angle, blade setting angle, blade length, flapping compensation coefficient, and horizontal hinge distance have a significant influence on the aerodynamic performance of the rotor system.

At present, various methods can be used to investigate helicopter rotor aerodynamics. Among them, the discrete vortex method is widely applied because of its relatively low computational cost, fast calculation speed, and suitability for studying unsteady aerodynamic problems. The basic idea of this method is to replace the rotor blade by a vortex sheet, which is then discretized into bound vortex segments on the blade and free vortex segments in the wake. By determining the circulation strength of these vortex segments, it is possible to calculate the pressure distribution, aerodynamic forces, moments, and corresponding aerodynamic coefficients of the rotor.

In this study, the discrete vortex method is applied to calculate the aerodynamic characteristics of coaxial helicopter rotors. A computational program is developed to analyze the influence of the initial azimuth angle and horizontal hinge distance on the thrust coefficient and the relative position of the upper and lower rotor systems. The obtained results provide a scientific basis for evaluating the aerodynamic characteristics of coaxial rotors, selecting appropriate geometric and kinematic parameters, and supporting the design, research, and teaching of helicopter aerodynamics in aviation training institutions.

II. METHODOLOGY AND COMPUTATIONAL PROGRAM

1. Introduction to FORTRAN

FORTRAN is one of the earliest and most widely used programming languages in the field of scientific and engineering computation. With its advantages in processing speed, capability to perform complex numerical calculations, and high stability in solving simulation problems, FORTRAN has been widely applied in mechanics, aerodynamics, aviation, and military engineering. In the aviation field, this language is commonly used to develop programs for calculating aerodynamic characteristics, structural analysis, motion simulation, and evaluating the operating parameters of aircraft and helicopters.

For the problem of calculating the aerodynamic characteristics of coaxial helicopter rotors, FORTRAN plays an important role in processing systems of linear algebraic equations, calculating induced velocity, vortex strength, pressure distribution, aerodynamic forces, and aerodynamic moments. The program developed in FORTRAN allows rapid variation of input parameters such as the initial azimuth angle, blade setting angle, blade length, flapping compensation coefficient, and horizontal hinge distance. Therefore, the program enables the investigation of the influence of geometric and kinematic parameters on the aerodynamic characteristics of the coaxial rotor system.

2. Development of a Program for Calculating the Aerodynamic Characteristics of Coaxial Helicopter Rotors

The theoretical basis of the program is the discrete vortex method. According to this method, the rotor blade is replaced by a vortex sheet. This vortex sheet is then discretized into bound vortex segments located on the blade and free vortex segments outside the blade. For each vortex segment, the circulation and vortex strength are assumed to be constant. Once the strength of the vortex segments is determined, the pressure distribution on the blade can be calculated, thereby allowing the determination of the corresponding aerodynamic forces, aerodynamic moments, and aerodynamic coefficients.

The computational problem is formulated based on several initial assumptions: the airflow around the rotor is considered an ideal, incompressible, and inviscid gas medium; the thickness of the lifting surface of the blade is neglected; and the external flow around the helicopter rotor satisfies the law of mass conservation and the continuity equation. The circulation of the vortex segments is determined through a system of equations established from the no-penetration boundary condition on the blade surface, the Chaplygin–Joukowski postulate, and the condition of circulation conservation along a closed contour.

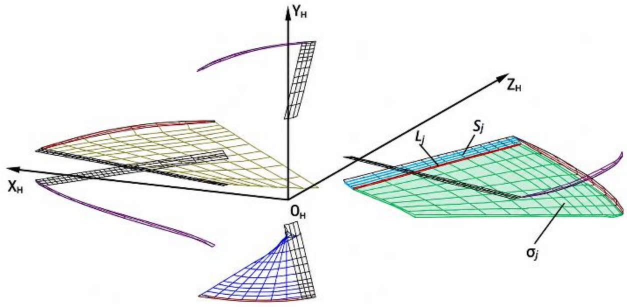


Figure 1. Blade discretization using the discrete vortex method

The computational program was developed to investigate the aerodynamic characteristics of coaxial helicopter rotors in two main cases:

- The influence of the initial azimuth angle on the thrust coefficient of the coaxial helicopter rotors.
- The influence of the horizontal hinge distance on the installation position of the two rotor stages of the helicopter.

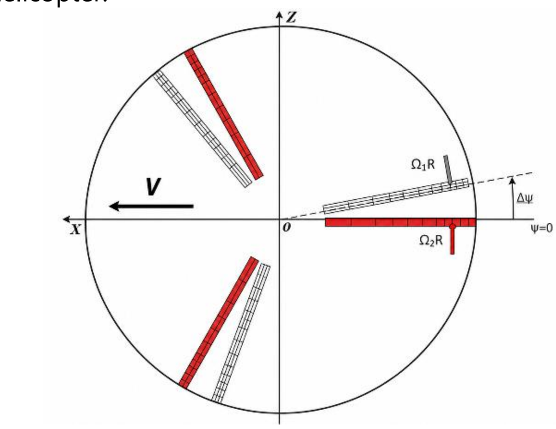


Figure 2. Initial azimuth angle Δψ

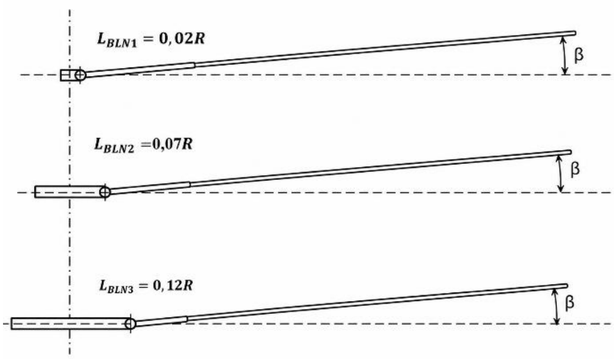


Figure 3. Horizontal hinge distance L_{BLN}

Based on the theoretical foundation of the discrete vortex method, together with the initial conditions and assumptions established for the problem, the airflow around the blade is considered an ideal, incompressible, and inviscid gas medium, while the thickness of the lifting surface is neglected. In addition, the external flow around the helicopter rotor is assumed to satisfy the law of mass conservation and the continuity equation. On this basis, an algorithmic flowchart was developed, and the FORTRAN programming language was used to build a computational program for determining the aerodynamic characteristics of coaxial helicopter rotors. The algorithmic flowchart is presented in Figure 4.

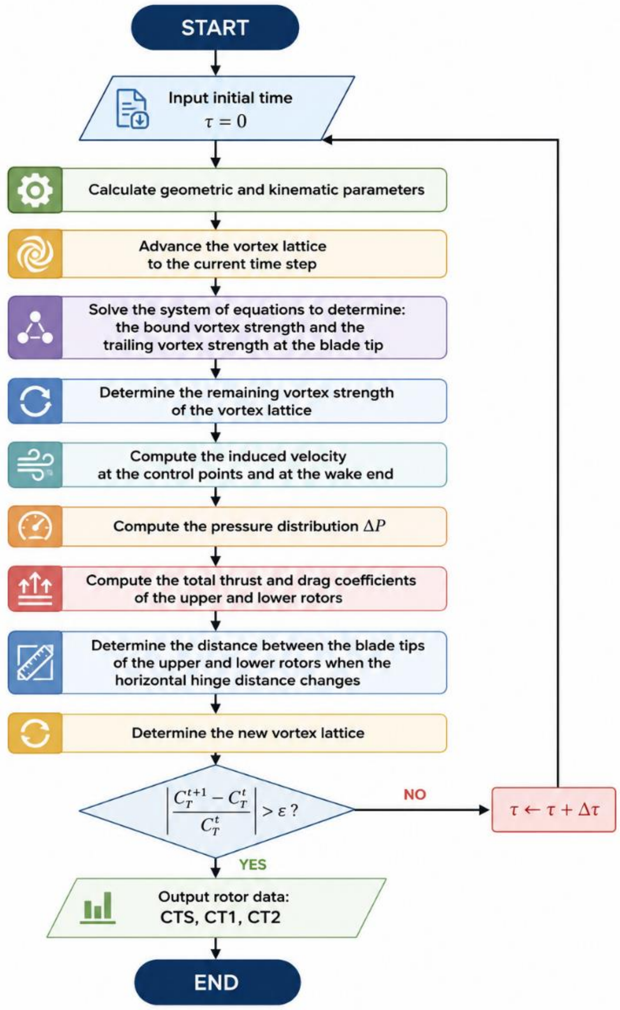
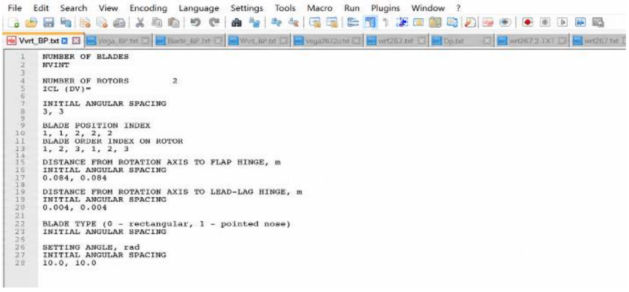


Figure 4. Flowchart of the FORTRAN computational program

The computational program was validated by comparing its results with available experimental data, showing good agreement. The program uses a set of input parameters that describe the kinematic and geometric characteristics of the rotor system, including the initial azimuth angle, blade setting angle, blade length, and flapping compensation coefficient. After processing these data, the program generates output parameters that characterize the aerodynamic behavior of the coaxial rotor, such as the thrust coefficient, side force, longitudinal force, and aerodynamic moments. These results provide a basis for analyzing and evaluating the aerodynamic characteristics of coaxial helicopter rotors.



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NUMBER OF BLADES
NVINT

NUMBER OF ROTORS        2
ICL (DV)=

INITIAL ANGULAR SPACING
3, 3

BLADE POSITION INDEX
1, 1, 2, 2, 2
BLADE ORDER INDEX ON ROTOR
1, 2, 3, 1, 2, 3

DISTANCE FROM ROTATION AXIS TO FLAP HINGE, m
INITIAL ANGULAR SPACING
0.084, 0.084

DISTANCE FROM ROTATION AXIS TO LEAD-LAG HINGE, m
INITIAL ANGULAR SPACING
0.004, 0.004

BLADE TYPE (0 - rectangular, 1 - pointed nose)
INITIAL ANGULAR SPACING

SETTING ANGLE, rad
INITIAL ANGULAR SPACING
10.0, 10.0
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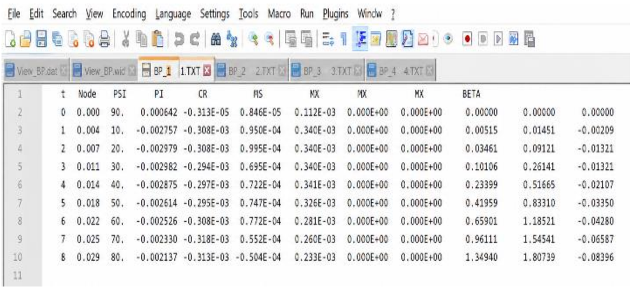
Figure 5. Input parameters of the computational program



t	Node	PSI	PI	CR	MS	MX	MX	MX	BETA		
0	0.000	90.	0.000642	-0.313E-05	0.846E-05	0.112E-03	0.000E+00	0.000E+00	0.00000	0.00000	0.00000
1	0.004	10.	-0.002757	-0.308E-03	0.950E-04	0.340E-03	0.000E+00	0.000E+00	0.00515	0.01451	-0.00209
2	0.007	20.	-0.002979	-0.308E-03	0.995E-04	0.340E-03	0.000E+00	0.000E+00	0.03461	0.09121	-0.01321
3	0.011	30.	-0.002982	-0.294E-03	0.695E-04	0.340E-03	0.000E+00	0.000E+00	0.10106	0.26141	-0.01321
4	0.014	40.	-0.002875	-0.297E-03	0.722E-04	0.341E-03	0.000E+00	0.000E+00	0.23399	0.51665	-0.02107
5	0.018	50.	-0.002614	-0.295E-03	0.747E-04	0.326E-03	0.000E+00	0.000E+00	0.41959	0.83310	-0.03350
6	0.022	60.	-0.002526	-0.308E-03	0.772E-04	0.281E-03	0.000E+00	0.000E+00	0.65901	1.18521	-0.04280
7	0.025	70.	-0.002330	-0.318E-03	0.552E-04	0.260E-03	0.000E+00	0.000E+00	0.96111	1.54541	-0.06587
8	0.029	80.	-0.002137	-0.313E-03	-0.504E-04	0.233E-03	0.000E+00	0.000E+00	1.34940	1.80739	-0.08396

Figure 6. Output parameters of the computational program

III. RESULTS OF THE COMPUTATIONAL PROGRAM

1. Influence of the Initial Azimuth Angle on the Thrust Coefficient of Coaxial Rotors

In this case, the program investigates the variation of the thrust coefficient when the initial azimuth angle between the upper and lower rotors is changed. Since the two stages of the coaxial rotor system rotate in opposite directions, the initial relative position between the rotor blades has a significant influence on aerodynamic interaction, induced velocity distribution, and the thrust coefficient of the entire rotor system. The input parameters include the initial azimuth angle $\Delta\psi$, flight speed, rotor rotational speed, blade setting angle, blade geometric dimensions, and parameters characterizing the flapping motion. The output results include the total thrust coefficient of the coaxial rotor system C_{TS}, the thrust coefficient of the upper rotor C_{T1} , and the thrust coefficient of the lower rotor C_{T2}

The dependence of the thrust coefficient on the initial azimuth angle can be expressed as:

$$C_{TS}=f(\Delta\psi);\ C_{T1}=f(\Delta\psi);\ C_{T2}=f(\Delta\psi)\#(1)$$

where C_{TS} is the total thrust coefficient of the coaxial rotor system; C_{T1} is the thrust coefficient of the upper rotor; C_{T2} is the thrust coefficient of the lower rotor; and Δψ is the initial azimuth angle between the two rotor stages.

From the output parameters, graphs can be constructed to show the influence of the initial azimuth angle on the thrust coefficient of the coaxial rotor, as well as the influence of the horizontal hinge distance on the installation position of the two rotor stages.

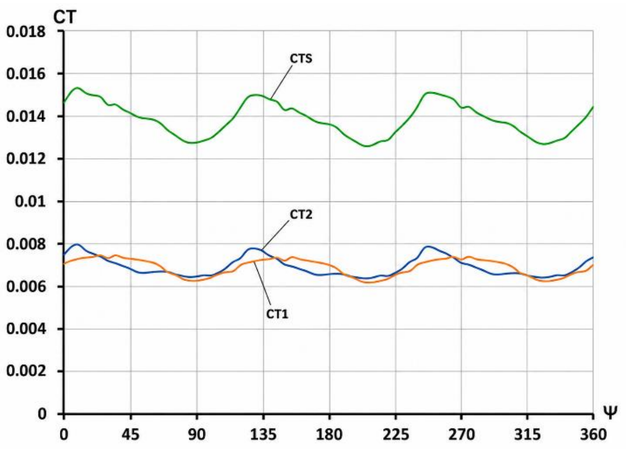


Figure 7. Influence of the initial azimuth angle on the thrust coefficient at $\Delta\psi=20^0$ (1)

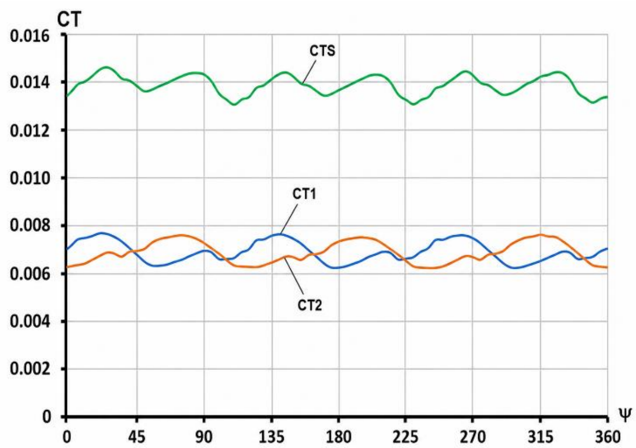


Figure 8. Influence of the initial azimuth angle on the thrust coefficient at $\Delta\psi=60^0$ (1)

This graph is constructed based on the computational results obtained from the program, in which the horizontal axis represents the initial azimuth angle Δψ, while the vertical axis represents the thrust coefficients C_{TS}, C_{T1}, and C_{T2}. Through this graph, the variation of the thrust coefficient can be evaluated when the initial position of the two rotor stages changes. This provides a basis for selecting a suitable initial azimuth angle to ensure that the thrust coefficient of the coaxial rotor system reaches an appropriate value during operation.

After the computational program had been developed, it was applied to investigate the aerodynamic characteristics of coaxial helicopter rotors in the two cases described above. The input parameters of the program include the geometric and kinematic quantities of the rotor system, such as the initial azimuth angle, blade setting angle, blade length, flapping compensation coefficient, flight speed, horizontal hinge distance, and other characteristic parameters. The program outputs the main parameters representing the aerodynamic characteristics of the coaxial rotor, including the thrust coefficient, side force, longitudinal force, and aerodynamic moments.

For the case investigating the influence of the initial azimuth angle, the computational results were used to construct graphs showing the dependence of the thrust coefficient on Δψ. As the initial azimuth angle changes, the relative position between the upper and lower rotors also varies, resulting in changes in the induced velocity field and the pressure distribution over the rotor blades. Consequently, the thrust coefficients of the upper rotor, the lower rotor, and the entire coaxial rotor system exhibit corresponding variations. These results indicate that the initial azimuth angle is an important parameter that should be considered in the analysis and selection of the operating configuration of coaxial rotors.

2. Influence of the Horizontal Hinge Distance on the Installation Position of the Two Rotor Stages

In the second case, the program investigates the influence of the horizontal hinge distance on the relative position between the upper and lower rotors. For coaxial helicopters, the spacing between the two rotor stages is of great importance, as it directly affects the clearance between the blade tips during rotation. This spacing also has a significant impact on the aerodynamic characteristics and operational stability of the entire rotor system.

When the horizontal hinge distance changes, the positions of the upper and lower blade tips also vary accordingly. Under the combined effects of blade flapping motion, flight speed, and aerodynamic interaction between the two rotor stages, it is necessary to determine the minimum clearance between the blade tips in order to ensure safe operating conditions.

The relationship between the spacing of the two rotor stages and the horizontal hinge distance can be expressed as:

$$h=f(l/R,V)h = f(l/R, V)h=f(l/R,V)\#(2)$$

where h is the relative distance between the tip of the upper rotor blade and the tip of the lower rotor blade; l/R is the horizontal hinge distance normalized by the rotor radius; and V is the helicopter flight speed.

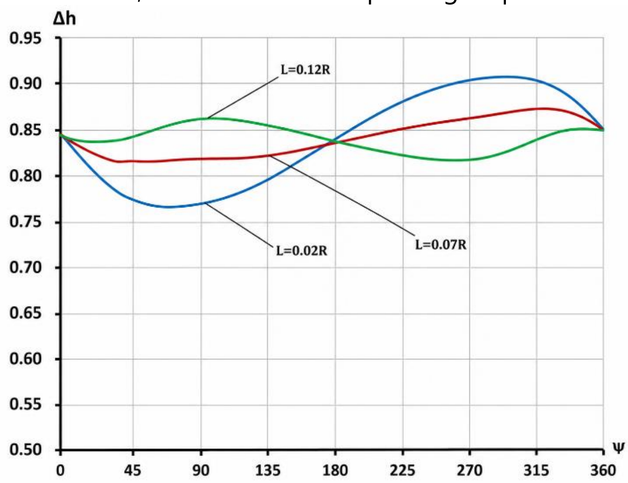


Figure 9. Influence of the horizontal hinge distance on the spacing between the two rotor stages at V_1=30.75 m/s (2)

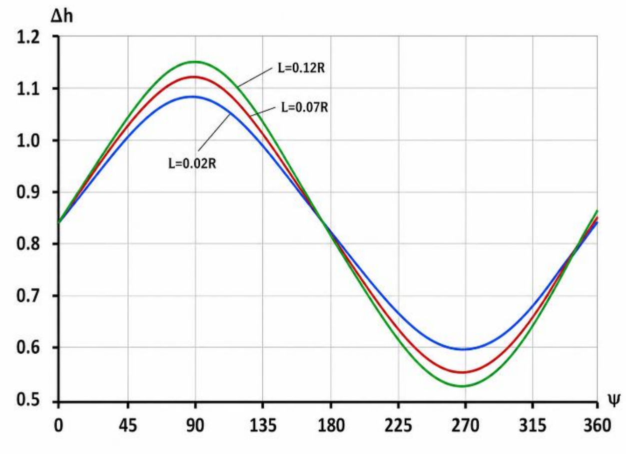
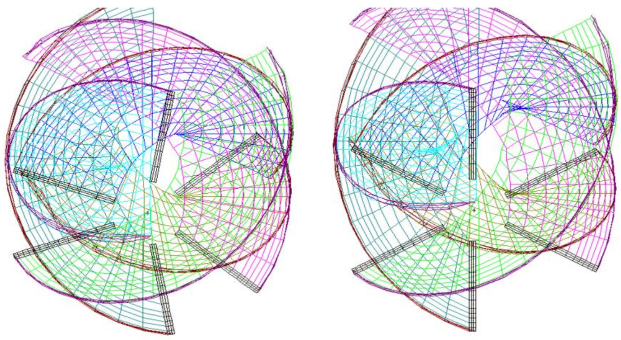


Figure 10. Influence of the horizontal hinge distance on the spacing between the two rotor stages at V_1=71.75 m/s (2)

The graph is constructed for different flight speeds, for example V_1=30.75 m/s and V_2=71.75 m/s. The horizontal axis represents the normalized horizontal hinge distance, while the vertical axis represents the clearance between the tips of the upper and lower rotor blades. The obtained results make it possible to evaluate the influence of the horizontal hinge distance on the installation position of the two rotor stages. This provides a basis for selecting appropriate geometric parameters to satisfy aerodynamic requirements and ensure structural safety.

For the case investigating the influence of the horizontal hinge distance, the program makes it possible to determine the clearance between the tips of the upper and lower rotor blades at different flight speeds. The results show that variations in the horizontal hinge distance lead to corresponding changes in the relative position of the two rotor stages. This provides an important basis for determining a suitable installation position for the upper and lower rotors, ensuring safe blade clearance during operation while maintaining appropriate aerodynamic characteristics.

In addition to the aerodynamic characteristic graphs, the program also enables visualization of the vortex wake structure behind the rotor. The vortex wake image illustrates the formation and development of free vortex segments in an unsteady flow field. When the effects of the initial azimuth angle or the horizontal hinge distance are considered, corresponding changes in the vortex wake structure can be observed. This indicates that the aerodynamic interaction between the upper and lower rotors has a direct influence on the flow field behind the rotor system.



a) V1=30,75 m/s; $\Delta\psi = 60^0$ b) V1=75,75 m/s; $\Delta\psi = 60^0$

Figure 11. Vortex wake structure under the effects of initial azimuth angle and horizontal hinge distance

IV. CONCLUSION

A computational program for determining the aerodynamic characteristics of coaxial rotors was developed using the FORTRAN programming language, which is widely applied in the aviation field for aerodynamic calculations of aircraft and helicopters. In this study, the program was used to investigate two factors affecting the thrust coefficient of coaxial rotors, namely the initial azimuth angle and the horizontal hinge distance, based on the discrete vortex method. After being developed, the program was verified and applied to solve the proposed problem. The results obtained by applying the discrete vortex method to unsteady flow show that variations in the input parameters, such as the initial azimuth angle and horizontal hinge distance, have a significant influence on the output parameters of the program, including the thrust coefficient and other quantities characterizing the aerodynamic behavior of coaxial rotors. These results provide a scientific basis for studying, calculating, and selecting suitable geometric and kinematic parameters of coaxial rotors in order to achieve an optimal thrust coefficient.

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