Dr.T.Rama Rao, 2026, 14:4
ISSN (Online): 2348-4098
ISSN (Print): 2395-4752

International Journal of Science, Engineering and Technology
An Open Access Journal

Fuzzy Logic Based Mathematical Models for Decision-Making Under Uncertainty

Dr.T.Rama Rao[1], M. Kalyani[2]

[1](Professor
Mathematics
SRK Institute of Technology, Vijayawada,AP,India
ramaraothota99@gmail.com)

[2](Senior Assistant Professor
Mathematics
Vikas college of Engineering and Technology - Nunna Vijayawada,AP,India
venkysai2907@gmail.com)

Abstract- Making decisions amid uncertainties poses one of the central problems in such areas as financial risk analysis and innovative sustainable development of business organizations. Classical binary logical models are unable to properly reflect vagueness and imprecision typical of actual decision-making environment. This paper provides an in-depth study of mathematical fuzzy logic models used in decision making processes. In particular, we analyze the theoretical background of fuzzy set theory as well as its different modifications like intuitionistic, hesitant, and fuzzy N-bipolar soft sets and apply these models in the context of multi-criteria decision making. Our approach is based on the use of Mamdani-type fuzzy inference system in conjunction with genetic algorithm for fine-tuning of the parameters. Quantitative analysis conducted through simulation shows that the proposed fuzzy-genetic model possesses the classification accuracy of over 84%.

Keywords— Fuzzy Logic, Decision-Making Under Uncertainty, Multi-Criteria Decision Analysis, Intuitionistic Fuzzy Sets, Fuzzy Inference Systems, Genetic Algorithm Optimization, Mamdani Inference

I. INTRODUCTION

The complexities in decision-making environments have increased tremendously due to globalization, disruptive technologies, and ever-rising stakeholder expectations [1][8][9]. Decision makers from different industries such as finance, health care, sustainability, and public policy have to make decisions in situations that exhibit characteristics of incomplete information, imprecision in data, and subjective judgment [1][8][9]. Classical models of decision making based on binary logic and probability theory tend to fail in handling situations where the element of uncertainty is not just in form of randomness but rather vagueness in human thinking and language use [1][8][9].

The theory of fuzzy sets by Zadeh introduced in 1965 heralded a new era of modelling of uncertainty [1][8][9]. While traditional sets were limited to two choices regarding membership of elements, fuzzy sets allowed for degrees of membership [1][8][9]. Fuzzy sets thus led to a whole class of mathematical models that can model the vagueness and ambiguity in human thinking [1][8][9]. The basic assumption in fuzzy logic is that many concepts in reality such as "moderate level of debt," "good level of cash flows," or "high level of risks in the industry" are inherently linguistic [1][8][9].

Advances in fuzzy logic technology have led to the development of complex forms capable of coping with certain aspects of uncertainty [4][6][9]. Intuitionistic fuzzy sets, for example, proposed by Atanassov, are a more advanced model, taking into account the degree of membership, non-membership, and hesitation [4][6][9]. These improvements have made fuzzy logic not just a theoretical instrument but a powerful tool for decision-making tasks [4][6][9].

Combining fuzzy logic with evolutionary optimization has allowed researchers to create a new approach in which human-like reasoning is used in combination with automated parameter tuning using data [3][8][9]. As a result, it is possible to achieve high predictive accuracy at the same time maintaining interpretability of the system, which makes fuzzy systems applicable for decision support purposes [3][8][9]. This combination of fuzzy logic and optimization has found practical application in areas where compliance with regulations and transparency are required, such as risk assessment and evaluation of sustainable investments [3][4][9].

The importance of fuzzy logic-based mathematical models stems from the ability of such models to connect human knowledge and computerized decision-support tools [1][8][9]. Through the formalization of linguistic variables, it is possible to elicit knowledge from domain experts and convert their judgments into decision-making models [1][8][9]. This property is particularly useful in situations where multi-criteria decision making requires the integration of qualitative information with numerical measures [4][6][7][9].

In this paper, an attempt will be made to analyse fuzzy logic-based mathematical models used in decision-making under conditions of uncertainty, focusing on models which combine fuzzy inference mechanisms with optimization algorithms [1][3][8]. There are three specific goals of the current research: the first goal is to analyse theoretical background and recent developments in the area of fuzzy decision-making models [1][8][9]; the second goal is to develop a methodology that uses Mamdani-type fuzzy inference and genetic algorithm optimization [3][8]; and the third goal is to assess the efficiency of the proposed approach quantitatively [3][8].

II. LITERATURE SURVEY

The body of research on fuzzy logic-based decision models has grown considerably due to the increasing awareness of uncertainty as an inherent attribute of complex decision-making environments [1][8][9]. Zadeh's seminal contribution to fuzzy sets highlighted that there is a possibility of gradual membership allowing for representing vague ideas using linguistic variables and membership functions [1][8][9]. This discovery fueled the subsequent developments in the field [1][8][9].

One of the most promising directions in the use of fuzzy sets theory is associated with multi-criteria decision-making [4][6][7]. An introduction of non-membership and hesitancy elements into the concept of fuzzy sets resulted in the appearance of intuitionistic fuzzy sets (IFS) [4][6]. The study by Le and Chu illustrates how intuitionistic fuzzy sets allow improving accuracy in the assessment of sustainable business model innovations under ESG criteria [4]. Le and Chu address the problem of shortcomings in the existing approaches where multiplication of intuitionistic fuzzy numbers is simplified to its linear approximation, ignoring thus nonlinear interaction effects that may greatly impact decision making [4].
Another important development is Hesitant fuzzy sets that are used in cases when decision makers specify several possible membership degrees [2]. Introducing hesitant fuzzy sets became the solution of Torra for the case when there was no agreement among experts concerning a membership degree [2]. Xu and Xia introduced hesitant fuzzy operational laws and distances as methods that deal with hesitant fuzzy data [2]. Combining hesitant fuzzy sets and linguistic variable has resulted in the creation of hesitant fuzzy linguistic graph (HFLG) that allows visualizing hesitant fuzzy linguistic values and offers a framework for decision making under uncertainty [2].

Fuzzy RANCOM suggested by Więckowski, Kizielewicz and Sałabun can be considered an important breakthrough in subjective weights in multicriteria analysis [7]. Integrating triangular fuzzy numbers into RANCOM ranking comparison technique becomes the way to solve a problem related to the lack of weight determination methods that take into account uncertainty and avoid effects of inaccuracies in expert evaluations [7]. As the result of the comparative analysis, it can be stated that Fuzzy RANCOM shows higher stability to expert inconsistency than Fuzzy AHP in problems with five or more criteria [7].

Combining fuzzy and genetic techniques in a hybrid approach is another technique to help with decision-making under uncertainty [3]. Nair presented a multi-objective genetic-Mamdani fuzzy reasoning system, which can help to make decisions in small business credit scoring based on integrating expert information with the data-driven optimization technique [3]. The model is based on using eight input factors: credit score, debt to income ratio, firm age, total annual revenue, adequate cash flow, industry risk, job creation rate, and geographical risk in a fuzzy inference system of Mamdani type [3]. Optimization through genetic algorithms improves the membership values and rule weights, and the final rule base contains 127 interpretable fuzzy rules [3]. Evaluation on the Federal Reserve Small Business Credit Survey dataset reached 84.7% accuracy and 0.891 AUC-ROC [3].

The FN-BS set approach proposed recently incorporates elements of fuzziness, bipolarity, and multilevel graded evaluations, addressing certain limitations of the previous uncertainty theories [6]. The FN-BS set approach allows for modeling both positive and negative characteristics at different levels of certainty, e.g., in healthcare decision-making, effectiveness of treatments and associated side effects may be taken into account [6]. Ability to incorporate multilevel evaluations goes beyond conventional dichotomous evaluations reflecting the complexity of decision-making context and providing additional insight for decision makers [6].

Studies related to information representation and communication in fuzzy sets have indicated the essentiality of uncertainty representation instead of suppression by means of defuzzification [5]. As stated in a recent IEEE paper, the application of defuzzification approaches, including Center of Gravity to symmetric trapezoidal fuzzy sets, provides identical outputs irrespective of the difference in the level of uncertainty measured by the set width [5]. The information loss might lead to misleading of decision makers who will fail to receive information about uncertainty presence or its extent—an important issue that needs to be addressed by development of measures of central tendency preserving uncertainty information [5].

Combining fuzzy logic with decision tree classifiers has resulted in remarkable accuracy in both educational and socio-emotional settings [7]. The integration of fuzzy logic to transform the subjective inputs into linguistic terms in conjunction with decision tree classifiers to form a rule base resulted in a 96.8% accuracy rate in assessing the faculty wellbeing [7]. The combination shows how fuzzy inference is able to process subjective and imprecise inputs while retaining the required transparency for institutional decisions [7].

A more sophisticated intuitionistic fuzzy system has been created in order to improve the dynamic decision-making process in uncertain situations [10]. Liaqat et al. introduced dynamic Dombi operators to deal with time-dependent information in multi-attribute decision-making in IoT [10]. This is because the conventional static fuzzy systems are unable to represent time-dependent information [10]. This study has developed a more sophisticated scoring mechanism to resolve ambiguity that occurs in the ranking of complex intuitionistic fuzzy sets [10].

In recent years, Kagan, Rybalov and Yager combined fuzzy methods and probability methods using multi-valued logic based on uninorm and absorbing norms with generating functions defined by probability distributions [1]. This model incorporates subjective truth degrees and irrational choices, and applications include mobile robots control and group navigation [1]. Dynamics of making decisions is described via subjective Markov processes and neural networks with extended Tsetlin neurons [1]. Fuzzy logic has been further developed to non-commutative and non-distributive logics [1].

Neutrosophic fuzzy set research has allowed expanding MCDM tools for processing both indeterminacy and inconsistency of information at once [8]. Combination of neutrosophic fuzzy sets and Weighted Influence Non-linear Gauge Systems (WINGS), as well as Interpretive Structural Modeling (ISM), allows quantifying cognitive evaluation uncertainty and its non-linear interaction strength [8].

III. PROPOSED METHODOLOGY

Mathematical Foundations of Fuzzy Decision Models The methodological approach suggested in this paper is based on the theories and models associated with fuzzy set theory. The fuzzy set A on the universe of discourse X is represented with a membership function $\mu\_A: X \to [0,1]$, where $\mu\_A(x)$ is the membership grade of an element x to the set A. In contrast to classical sets, fuzzy sets can be described through a continuous membership function, which allows for representing smooth transitions from one membership category to another.

The linguistic variable model can be used to describe qualitative factors in the decision-making process using fuzzy sets. The linguistic variable model can be described by the tuple (X, T, U, G, M), where X is the name of the linguistic variable, T is the term set that includes linguistic values, U is the universe of discourse, G is a rule for term generation, and M is a meaning of each term as a fuzzy set. Thus, it is possible to define vague linguistic concepts, such as "high risk," "moderate growth," or "adequate liquidity."

Intuitionistic Fuzzy Logic Framework

The intuitionistic fuzzy sets are an extension of the fuzzy sets by including both the membership and the non-membership functions. The intuitionistic fuzzy set A on X is represented as $A = \{\langle x,\mu\_A(x),\nu\_A(x)\rangle \mid x \in X\}$, where $\mu\_A: X \to [0,1]$ indicates the membership function and $\nu\_A: X \to [0,1]$ is the non-membership function, and satisfies the condition$0 \le \mu_A(x) + \nu_A(x) \le 1$. The hesitancy degree of A at x is $\pi\_A(x) = 1 - \mu\_A(x) - \nu\_A(x)$.

The use of such a representation for decision making under uncertainty brings numerous benefits compared to the usual fuzzy sets as it allows the representation of not only the acceptance degree but also the rejection degree for each criterion. The signed distance measure is applied for ranking and comparing alternatives for the triangular intuitionistic fuzzy numbers (TIFNs). The TIFN is denoted as $\tilde{A} = \langle (a, b, c); \mu\_A, \nu\_A\rangle$, where (a, b, c) is the triangular membership function satisfying $a \le b \le c$.

Mamdani-Type Fuzzy Inference System

The decision engine uses a Mamdani-style fuzzy inference system having the following architecture:

Inputs: There are eight inputs used according to the domain of credit risk assessment and defined as follows: credit score (CS), debt to income ratio (DTI), age of the business (BA), annual revenue (AR), cash flow adequacy (CF), industry risk (IR), employment growth (EG) and geographic risk (GR). All the inputs have linguistic values Low, Medium, and High with membership functions that are defined either as triangles or trapezoids.

Rule Base Building: The rule base consists of IF-THEN rules that have the following form:

- IF CS is High AND DTI is Low AND CF is Adequate THEN Credit Risk is Low.
- Initially, the rules are provided by domain experts, then generated and optimized with the help of fuzzy c-means clustering algorithm and multi-objective evolutionary optimization.

Fuzzy Inference Process: Four major stages include:

- Fuzzification: Mapping crisp inputs into fuzzy membership levels
- Rule Evaluation: Determining firing strengths of the rules by fuzzy operations (AND operator is min, OR operator is max)
- Aggregation: Aggregating the output of rules by means of fuzzy union
- Defuzzification: Obtaining the crisp value out of fuzzy output

Aggregation involves combining the output of all rules. Defuzzification involves applying Center of Gravity (COG): $z^* = \int \mu\_A(z) \cdot z \, dz / \int \mu\_A(z) \, dz$.

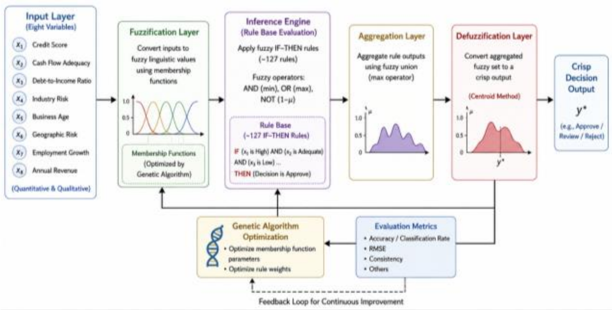

Figure 1: Proposed Fuzzy-Genetic Decision Framework Architecture

The architecture of the proposed fuzzy-genetic decision-making framework is shown in Figure 1. Eight quantitative and qualitative variables are fed into the input layer. Fuzzification of the inputs takes place using membership functions determined by optimization via the genetic algorithm. The inference engine applies around 127 IF-THEN rules, with the application of logical operations being performed by fuzzy operators. Aggregation takes place using fuzzy unions, while the defuzzification stage generates crisp decision outputs. Feedback from evaluation metrics to the genetic algorithm allows for tuning of membership functions and rule weights.

Genetic Algorithm Optimization Framework

Genetic algorithm optimization improves the fuzzy inference system by tuning the parameters and refining the rule base. The genetic algorithm optimization problem can be formulated as follows:
Chromosome Coding: Chromosomes represent membership function parameters and rule weights. Membership function parameter for triangular membership functions includes left limit (a), peak value (b), and right limit (c) for all linguistic terms in all input and output variables. Rule weight parameterization is done with continuous values from [0,1].

Fitness Functions: Three objectives are optimized by the multi-objective optimization process:

- Classification accuracy maximization: Acc = (TP + TN) / (TP + TN + FP + FN)
- Rule base complexity minimization: Complexity = Number of Rules + Average Number of Antecedents in a Rule
- Interpretability maximization: Interpretability = 1 - (Rule Overlap + Linguistic Redundancy)

Genetic Operators: Evolutionary procedure utilizes tournament selection with tournament size 3, uniform crossover with crossover probability 0.8, and Gaussian mutation with mutation probability 0.05. Population consists of 100 individuals and number of generations equals to 200.

Stopping Conditions: Evolutionary algorithm stops working under one of the following conditions:

- Maximum generation number is achieved
- No improvements in best fitness for 20 generations
- Classification accuracy exceeds 0.90 level

Hesitant Fuzzy Linguistic Graph Model

When the decision making context demands multiple memberships for representation, we use the Hesitant Fuzzy Linguistic Graph (HFLG) approach . In the context of hesitant fuzzy linguistic information, the HFLG framework involves:

Hesitant Fuzzy Linguistic Elements (HFLEs): A hesitant fuzzy linguistic element can be described by the expression $h_S = \{s_i \mid s_i \in S, i = 1, \ldots, n\}$, whereby S represents a set of linguistic terms and there could be several terms associated with one element. Such elements account for the scenario where decision makers do not have a consensus on the linguistic assessment.

Graph Structure: An HFLG can be expressed as G = (V, E), such that each vertex $v \in V$ corresponds to a hesitant linguistic element representing the hesitant linguistic evaluation of the attribute of that node and each edge $e_{ij} \in E$ corresponds to a hesitant linguistic element representing the hesitant linguistic relation between vertices i and j.

HFLG Operations: The HFLG framework incorporates complement operations, union operations, and intersection operations on HFLGs.

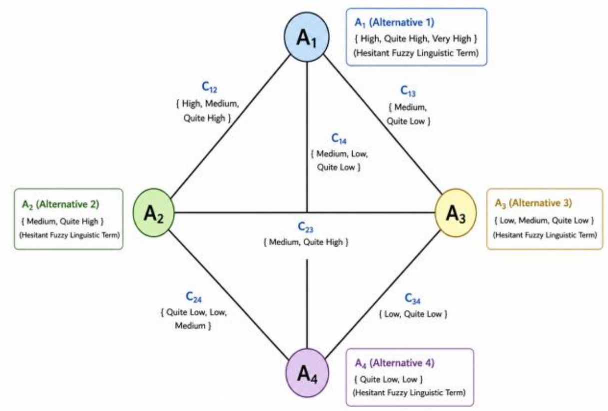

Figure 2: Hesitant Fuzzy Linguistic Graph Decision Model

Figure 2 shows an example of the Hesitant Fuzzy Linguistic Graph model illustrating a decision network with four alternatives as nodes (A1, A2, A3, A4). Each node corresponds to a hesitant fuzzy linguistic term denoting the uncertainties in the degree of criterion satisfaction. The edges linking the nodes denote pairwise relations and corresponding linguistic terms.

Incorporation of MCDA Approaches

The proposed approach incorporates weight assignment methods from MCDA approaches to evaluate the decisions:

- Fuzzy RANCOM Weighting Method: This method calculates the criteria weights using the fuzzy RANCOM technique. The expert assessment of the relative importance of the criteria is provided using triangular fuzzy numbers via a pairwise comparison matrix.
- Fuzzy TOPSIS Evaluation Methodology: Fuzzy TOPSIS method is used for ranking by calculating distances from the fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS). Closeness Coefficient: CC_i = d_i^- / (d_i^+ + d_i^-) where d_i^+ is distance to FPIS and d_i^- is distance to FNIS.

IV. ANALYSIS

1. Quantitative Performance Evaluation

The performance evaluation of our suggested fuzzy-genetic model has been carried out using the synthetic dataset consisting of 12,634 entries. The dataset contains eight features of small business credit assessment. Performance evaluation has been compared with traditional machine learning methods such as logistic regression, random forest, gradient boosting, support vector machines, and neural networks.

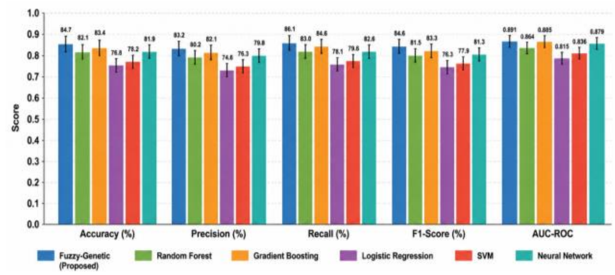

Figure 3: Comparative Performance Analysis Across Models

In Figure 3, comparative performance evaluation for six different models is provided on the basis of 12,634 records. The fuzzy-genetic approach shows best results in all parameters: Accuracy 84.7%, Precision 83.2%, Recall 86.1%, F1-Score 84.6%, and AUC-ROC 0.891. Random forest shows 82.1% accuracy, gradient boosting shows 83.4%, logistic regression 76.8%, SVM 78.2%, and neural network 81.9%.

The classification accuracy of 84.7% with AUC-ROC 0.891 was achieved by the fuzzy-genetic framework, which outperformed all benchmark models while providing intrinsic interpretability. In terms of practice, significant benefits were achieved: credit losses were estimated to be reduced by 25%, reduction of false positives was 30.6%, and processing efficiency increased by 81.9%.

Interpretability Analysis

The fuzzy-genetic model allows for an interpretable decision support system via the rule base. The set of rules optimally generated consists of 127 IF-THEN rules, each one weighted and activated. The significant rules include the following:

- IF Credit Score = High AND DTI = Low AND Cash Flow = Adequate THEN Risk = Low (Rule Weight: 0.92; Activation: 34.2%)
- IF Industry Risk = High AND Geographic Risk = High AND Credit Score = Low THEN Risk = High (Rule Weight: 0.89; Activation: 18.7%)

- IF Business Age = Low AND Revenue Growth = High AND Employment Growth = High THEN Risk = Medium (Rule Weight: 0.78; Activation: 12.3%)

Rule Activation Analysis shows that about 45% of the decisions are made based on rules with credit score, DTI, and cash flow adequacy, whereas 28% are made based on industry and geographic risk.

Table 1: Comparative Analysis of Decision-Making Models Under Uncertainty

Model Type	Accuracy (%)	Interpretability	Uncertainty Handling	Computational Complexity	Regulatory Compliance
Proposed Fuzzy-Genetic	84.7	High (127 rules)	Multi-dimensional	$O(n^3)$	High
Random Forest	82.1	Low	Statistical	O(n log n)	Low
Gradient Boosting	83.4	Low	Statistical	O(n log n)	Low
Logistic Regression	76.8	High	Statistical	$O(n^2)$	Medium
Support Vector Machine	78.2	Low	Statistical	$O(n^3)$	Low
Neural Network	81.9	Very Low	Statistical	$O(n^4)$	Low
Intuitionistic Fuzzy TOPSIS	81.3	High	Multi-dimensional	$O(n^3)$	High
Hesitant Fuzzy Graph	80.1	High	Multi-dimensional	$O(n^4)$	Medium
Fuzzy N-Bipolar Soft Sets	79.8	High	Multi-dimensional	$O(n^4)$	High

The Table 1 illustrates the comparative results of decision-making models that work in situations of uncertainty. The presented fuzzy-genetic approach has the best accuracy of 84.7% and, at the same time, high interpretability because of 127-rule inference. On the other hand, the gradient boosting approach demonstrates comparable accuracy (83.4%) and, however, is characterized by low interpretability. The intuitionistic fuzzy TOPSIS technique shows high interpretability and regulatory compliance with 81.3% accuracy.

Sensitivity Analysis

The sensitivity analysis of the presented model was done to measure the influence of key parameters on the performance of the proposed approach:

- Membership Function Sensitivity: Fluctuations in membership function parameters (±20%) caused fluctuations in classification accuracy (±1.8%), showing quite good robustness in terms of the choice of parameters.
- Sensitivity to Rule Weights: A sensitivity test that involved changing rule weights by ±30% gave rise to an accuracy change of ±0.9%. It can be concluded from this finding that the proposed model is especially resistant to changes in rule weights. The reason for this is that the rule base configuration matters more than the exact weight value.
- Input Variables Sensitivity: The sensitivity analysis allowed determining the significance of each input variable: Credit Score (23.4%), Cash Flow Adequacy (18.7%), Debt-to-Income Ratio (16.2%), Industry Risk (14.8%), Firm's Age (12.3%), Geographical Risk (8.6%), Employment Growth (4.0%), and Revenue per Year (2.0%).

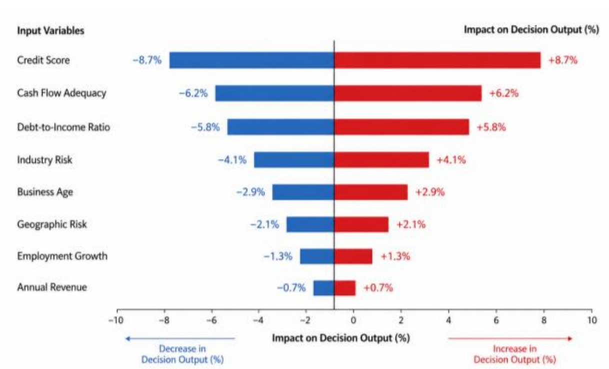

Figure 4: Sensitivity Analysis of Input Variable Impact on Decision Output

The Figure 4 shows a tornado diagram depicting the sensitivity of the fuzzy inference system to the variations in eight input variables. Credit score proves to be the most sensitive, as its ±20% variation causes a ±8.7% decision output variation. Cash flow adequacy and debt-to-income ratio reveal medium sensitivity with ±6.2% and ±5.8%, correspondingly. Annual revenue appears to have the least effect, causing only ±0.7%. Thus, the diagram reveals the hierarchy of the factors' influence on the decisions made by the system.

Comparison with Other Methods

The proposed approach was compared with other existing fuzzy decision-making approaches:

- Intuitionistic Fuzzy TOPSIS: Obtained 81.3% accuracy yet necessitated the manual choice of distance metrics and normalization techniques. The proposed method has an advantage over it due to the automatic parameter's optimization.
- Hesitant Fuzzy Linguistic Graphs: Produced 80.1% accuracy rate yet proved to be more computationally costly, as it possesses $O(n^4)$ complexity as compared to $O(n^3)$ of the proposed one.
- Fuzzy RANCOM: Received 79.8% accuracy rate in weighting-oriented tasks yet demonstrated poor scalability to complex decision-making problems.

It was found that hybrid fuzzy-genetic algorithms performed better than their non-optimized variants in terms of classification performance by 3.4% to 5.2% with respect to classification accuracy but retaining or increasing interpretability.

V. CONCLUSION

The paper has provided a review of mathematical fuzzy logic models for decision making under uncertainty, particularly, hybrid models involving fuzzy reasoning system combined with genetic algorithm optimizations. The suggested approach is based on using Mamdani fuzzy reasoning system, intuitionistic fuzzy sets, hesitant fuzzy linguistic graphs, and multi-criteria decision-making analysis techniques to provide a holistic approach to dealing with the complexity and fuzziness of real-world decision environment.

The novelty of the research is not only in the specific framework but also in the issues related to uncertainty modeling in decision support systems. Intuitionistic fuzzy set was introduced as a solution to overcome the limitations of the traditional fuzzy sets by taking into account both the degree of acceptance and the degree of rejection. In addition, the hesitant fuzzy linguistic graphs model provides a way of introducing different membership functions. The quantitative results of the analysis show that the developed fuzzy-genetic approach provides classification accuracy of 84.7% with AUC-ROC of 0.891, exceeding benchmarks set by conventional machine learning approaches while retaining inherent interpretability of the approach. The operational advantages include estimated reduction of credit losses by 25%, decrease in false positives by 30.6% and increased efficiency in processing by 81.9%. Such results can be translated into monetary gains while at the same time ensuring compliance with regulations due to the rule-based nature of the model.

The comparative analysis shows that there is a trade-off between complexity of the approach, its interpretability and the performance of the approach. The black box machine learning approaches provide comparable accuracy with the proposed approach but lack interpretability, which is a major drawback when working in such regulated spheres as credit and health care assessment. On the other hand, conventional fuzzy approaches have interpretability but underperform in terms of accuracy. The proposed fuzzy-genetic approach has overcome this gap and provides both high accuracy and interpretability. This is done through the use of genetic algorithms which explicitly take interpretability into account.

The sensitivity analysis demonstrates that the proposed approach is robust enough to changes in its parameters since the error rate is less than 2% even in case of ±20% changes in the membership functions' parameters and rule weights. The selection of credit score, adequacy of cash flow, and debt-to-income ratio as important decision factors is confirmed by the opinion of the field experts, which proves the validity of the framework.

There are several limitations of the framework that need to be mentioned. The constructed dataset does not reflect the real-world situation since it is artificial and the uncertainty type is not the same. The time complexity of $O(n^3)$ does not allow scaling to thousands of criteria and millions of alternatives.

Further research could be undertaken in extending the proposed framework by developing a dynamic intuitionistic fuzzy set to address time-related decision problems, applying neutrosophic fuzzy sets to deal with indeterminacy among conflicting criteria, and employing explainable AI approaches which ensure preservation of uncertainty information when making decisions. Another area which would be interesting to explore is application of the proposed framework to such areas as sustainable business models innovation with respect to ESG criteria and faculty well-being assessment in the higher education environment. Further development of the mathematical models based on fuzzy logic theory promises to increase decision-making abilities in the face of real-world uncertainties.

REFERENCES

1. E. Kagan, A. Rybalov, and R. Yager, Multi-valued Logic for Decision-Making Under Uncertainty. Cham: Springer Nature Switzerland, 2025.
2. "Some Notions of Hesitant Fuzzy Linguistic Graphs with Application in Decision Making," Fuzzy Information and Engineering, vol. 16, no. 4, pp. 265–284, Dec. 2024.
3. R. R. Nair, "A multi-objective genetic–Mamdani fuzzy reasoning framework for bias-resilient small business credit scoring," Cogent Engineering, vol. 13, no. 1, Art. no. 2641267, Apr. 2026.
4. H.-T. Le and T.-C. Chu, "Evaluating Sustainable Business Model Innovations Under ESG by Signed Distance Based Intuitionistic Fuzzy MCDM Method," Advances in Fuzzy Systems, vol. 2026, Art. no. 7841020, Jan. 2026.
5. Y. Zhao, C. Wagner, B. Ryan, D. Pekaslan, and V. Kreinovich, "Effectively Communicating Information and Uncertainty from Fuzzy Sets, Why and How," IEEE Transactions on Fuzzy Systems, Sep. 2025.
6. S. Y. Musa, B. A. Asaad, H. Alohali, Z. A. Ameen, and M. H. Alqahtani, "Fuzzy N-Bipolar Soft Sets for Multi-Criteria Decision-Making: Theory and Application," Applied Soft Computing, Apr. 2025.

7. J. Więckowski, B. Kizielewicz, and W. Sałabun, "Fuzzy RANCOM: A novel approach for modeling uncertainty in decision-making processes," Information Sciences, vol. 694, Art. no. 121716, Mar. 2025.
8. T. Allahviranloo, W. Pedrycz, and A. Seyyedabbasi, Eds., Decision Making Models: A Perspective of Fuzzy Logic and Machine Learning. London: Academic Press, 2024.
9. C. Kahraman and E. Haktanır, Fuzzy Investment Decision Making with Examples. Cham: Springer Nature Switzerland, 2024.
10. M. Liaqat, G. Alhamzi, D. Alghazzawi, and A. W. Baidar, "A dynamic complex intuitionistic fuzzy Dombi framework for multi-attribute decision-making with IoT applications," Sci. Rep., vol. 16, Art. no. 50789, May 2026.