Pushpandra Kumar, 2026, 14:4
ISSN (Online): 2348-4098
ISSN (Print): 2395-4752

International Journal of Science, Engineering and Technology
An Open Access Journal

Performance Evaluation and Application of Discrete Time Queueing Models for Computer and Communication Networks

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Abstract- In this article, the study of discrete time queueing models for computer communication networks and their applications to queueing models. It is expected that a discrete time model is "slotted". During a slot, a discrete time queue may accept and service a maximum of one packet each. The major reason of discrete time queueing systems have attracted interest of potential applications that take use of slotted time. Comparing how continuous time models are handled, we take into consideration several conclusions inside a single discrete-time framework. This method has two benefits: it parallels the treatment of continuous time models as much as feasible, and It streamlines the handling of several discrete-time models under a single structure. This enables us to compare and contrast the continuous and discrete time queues. Specifically, Our applications related to birth and death center on the discrete-time single server model with bernoulli arrivals and geometric service times. We give the reader a straightforward, meticulous analysis covering every one of the five scheduling guidelines examined inside the written works, focusing especially on the stationary distributions at the pre arrival, slot centers, and slot margins.

Keywords— Discrete time models, Queue, Communication Systems.

I. INTRODUCTION

The major focus of this article is on discrete time queueing models that match the discrete formulas for birth and death in order to provide as a summary of discrete-time queues. In addition to providing the fixed distribution at the centers and edges of slots, and pre arrival moments, we cover the majority of scheduling rules found in the literature. We also talk about Markovian models that can be solved recursively with ease. In particular, we discuss the Markovian models with many servers, batch arrival, and finite population, Choose models that allow for the solution of their stationary distribution using global balancing equations. In addition, we concentrate on transform-free techniques to simplify the analysis and provide several conclusions in a single framework for discrete time. This method has two benefits: it parallels the treatment of continuous time models as much as feasible, and it brings several discrete-time concepts under one framework for treatment.

Continuous time queueing systems have been studied since Erlang's early work (Erlang 1909) .However, Discrete time queueing systems is a rather distinct paradigm that has garnered more attention lately.

It is believed that time discrete time models are "slotted," or that it is made up of a fixed-length concatenation periods known as slots. Activities are limited to occurring during these times. For example, in a discrete time queue, a maximum of one packet may be accepted and serviced within a slot. There might be several events that take place during each slot on the whole network.

Potential applications that leverage slotted time have been the main source of interest in discrete time queueing systems. Thus, a fundamental aspect of the architecture and related analysis is the slot the amount of time it takes to send a packet of a particular length.

Three case studies concerning high-speed networks are included in this paper's conclusion; these kinds of issues create a new difficulty for the application of queueing models. This case study is valuable because it integrates concepts related to non-markovian queues, networks of queues, and transform methods inside a real-world scenario.

II. UTILIZATION IN COMPUTER SYSTEMS

Computer systems often have queues. Interaction between computer systems to respond to various questions pertaining to the queues inquiry (Allen 1977). Computer systems with queuing systems make it easier to compute service facilities with one or more servers. Additionally, buffers with finite and infinite capacity can benefit from it. such as waiting rooms. Diverse individuals from various demographic groups attempt to get services by using the queueing system. Furthermore, inside a computer system, the term "customer" can refer to any form of inquiry or request within the system, a task within the system, a packet within a communication network, or a computer program (Inria 2004) .

A specific consumer exits the queueing system after being serviced. When service centers are overbooked, customers are directed to the computer waiting system.

a/ b/ c/ K

In this notation, the inter arrival time distribution is represented by a. In this concept, b stands for the arrangement of service hours, c for the number of servers, and K for the total capacity of the system, which includes the servers .

Because the letter M reflects the exponential distribution provided by Markov, it frequently replaces the symbol of a,(Bolch et al.2006). Conversely, G or GI stands for generic distribution, and D for deterministic distribution. Many computer system applications result in various customer classes receiving preferred treatment, meaning that clients with higher priority are treated first and services are delivered in a queue.

One of the best techniques for performing the queue analysis for a Markov chain is the queue length distribution. Process is a Markovian discrete set of points on the time axis. see in (Medhi,2003) . Using a Markov process, the probability function for the relevant random variable is established. The amount of time that passes between a job's departure and arrival is used in computer systems to determine a job's reaction time. The period of time is referred to as "turn around time," especially in the computer sector. Knowledge of queueing theory increases the ability of the applied mathematician and system analyst to communicate with each other.

III. USE IN COMMUNICATION SYSTEMS

Communication systems is another area where queueing theory is applied via the Markov process.see in (Giambene, 2014) . The past, present, and future are all regarded as separate entities for the sake of this chain of operations. One other example of how queueing theory is used in Systems of communication are the natural rule of sequence of jumps implemented inside the Markov chain procedure. see in (Busson,2012) .

IV. SYSTEMS OF DISCRETE TIME QUEUING

Time has been taken to be continuous in most queueing mechanisms that have been covered thus far. In other words, arrivals and departures can happen at any time. Some real-world systems function differently, using discrete time. Events in this case might only be permitted to happen at regular, equispaced instants.

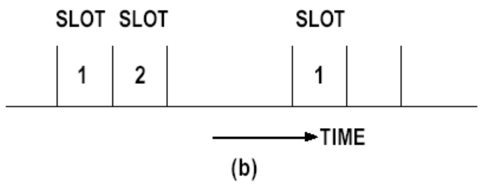

Fig. 1.1 : Two station slotted network

With probability "a" a packet will reach a station across a time period. We'll suppose that there is just one packet that each station can carry. Additional packet arrivals to a station are prevented after a packet reaches there. A station broadcasts a packet with probability "s" during each slot if it has one. Only one station may successfully broadcast a packet during a slot. if a packet is not transmitted simultaneously by the other station. Packets overlap (or "collapse") and get jumbled if they are sent by both stations in the same slot. in order for neither to be effectively transferred. Later, both packets need to be retransmitted. Every station is able to identify when one of these collision events occurs.

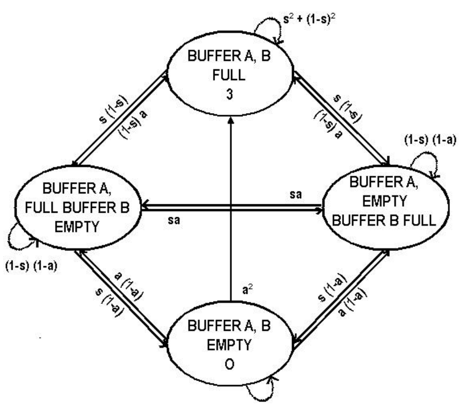

Fig. 1.2 : Two station state transition diagram

The system may enter state 3 should both buffers be empty and each receives a message with probability a^2. The system will stay in the same state if either a collision occurs (with probability s^2) or not a single transmission attempt (with probability 〖(1-s)〗 ^2). It is possible station B will send its package at that time. successfully with a probability of s(1-s) and station A will transmit successfully with a probability of s(1-a). For any state J of N states, global balance equations may be written for this type of discrete time Markov chain, where p_ij represents the probability of moving from state i to state j and π_j denotes the likelihood of state j's equilibrium:

$$\pi_j \sum_i p_{ji} = \sum_i \pi_i p_{ij}, \qquad (1.1)$$

$$\pi_j = \sum_i \pi_i p_{ij}, \qquad j = 1,2,....,N \qquad (1.2)$$

Naturally

$$\sum_i p_{ji} = 1, \ \sum_i \Pi_i = 1 \qquad (1.3)$$

Equation (1.2) may be expressed in matrix form as

$$\underline{\pi} = \underline{\pi}\ \underline{P} \qquad (1.4)$$

Where ‗π =[π_0,π_1,π_2......] and the matrix of transition is P .In other words. i.e, the entry of ijth elements of P is pij.

An alternative, more computationally efficient method of solving the problem starts with an estimate of the state probabilities at slot 0 that is likely to be off. From there, it follows the equilibrium probability vector. The recursion may then be used to generate the state probabilities at each slot.

$$\underline{\pi}^{(n)} = \underline{\pi}^{(n-1)} \underline{P} \qquad (1.5)$$

Recursion will converge to equilibrium state probability if it is allowed to continue for a long time. A convergence test like

$$|\pi_i(n) - \pi_i(n-1)| / \pi_i(n-1)| \le \varepsilon \text{ for each } i \qquad (1.6)$$

Here, the positive value ε is significantly less than 1.

As is frequently the case, functions of the state probabilities may be used to compute performance metrics for the basic model shown in figure 1.2. For example, a quick traversal will reveal what the channel's typical throughput is.

Mean throughput =π_1 (s)+π_2 (s)+π_3 (2s(1-s))

$$|\pi_i(n)-\pi_i(n-1)|/\pi_i(n-1)|\le \varepsilon \text{ for each } i \quad (1.7)$$

The two packet buffers have an average of the following number of packets waiting:

Mean # of packets = $\pi_1+\pi_2+2\pi_3$ (1.8)

The average latency for packets that are successful while stepping into a buffer, calculated using Little's law, is

$$\text{Mean delay}=\frac{\pi_1+\pi_2+2\pi_3}{\pi_1(s)+\pi_2(s)+\pi_3(2s(1-s))} \quad (1.9)$$

Lastly, the likelihood that an incoming packet would be prevented from going into a buffer is known as the blocking probability.

$$0.5\pi_1+0.5\pi_2+\pi_3 \quad (1.10)$$

Remember that when examining N stations spread over several access systems, the number of terminals that have packets ready to broadcast is usually utilized as the state variable.

V. THE QUEUEING SYSTEMS

$Geom/Geom/1$ **and** $Geom/Geom/1/N$ The $M/M/1/N$ continuous time queueing system's discrete time counterpart is this one. Depending on whether a new client may be served in the same slot as an existing one, there are really two types of $Geom/Geom/1/N$ queuing systems that can be investigated. A system that allows a client to be served in the same slot even if there is no line when they arrive, by (Badr et al.1986). The $Geom/Geom/m/N$ queueing model where m=1, or Using only basic concepts, one may determine that this system's transition probabilities are

$a_{0,0}$ = ps + (1-p) (1.11)
$a_{n,n}$ = ps + (1-p) (1-s) (1.12)
$a_{N,N}$ = p(s + (1-s)) + (1-p) (1-s) (1.13)
$a_{n,n+1}$ = p(1-s), n = 0,1, ….., N-1 (1.14)
$a_{n,n-1}$ = (1-p) s, n = 1,2,…..N (1.15)

The state transition diagram of the queueing model is topologically similar to the queueing system, except for transition probabilities from every state to itself. One can construct vertical barriers between neighboring states and equalize the flow of probability flux over the border in both directions to solve for the probabilities of the equilibrium states. Assuming to be the equilibrium state probability, the queueing system has i customers.

p(1-s) P_0 = s(1-p) P_1 (1.16)
p(1-s) P_1 = s(1-p) P_2 (1.17)
p(1-s) P_2 = s(1-p) P_3 (1.18)
p(1-s) P_{n-1} = s(1-p) P_n (1.19)
p(1-s) P_{N-1} = s(1-p) P_N (1.20)

It is simple to express the recursion using these equations:

$$P_1=\frac{p(1-s)}{s(1-p)}P_0 \quad (1.21)$$

$$P_2=\frac{p(1-s)}{s(1-p)}P_1, \quad (1.22)$$

$$P_3=\frac{p(1-s)}{s(1-p)}P_2 \quad (1.23)$$

$$P_n=\frac{p(1-s)}{s(1-p)}P_{n-1} \quad (1.24)$$

$$P_N=\frac{p(1-s)}{s(1-p)}P_{N-1} \quad (1.25)$$

When one is substituted for the other, the compact expression results.

$$P_n=\left[\frac{p(1-s)}{s(1-p)}\right]^n P_0 \qquad n=1,2,\ldots\ldots N \quad (1.26)$$

Using the using probability conservation to find P0 yields

$$P_0 = 1/\sum_{i=0}^{N}\left[\frac{p(1-s)}{s(1-p)}\right]^{l} \qquad (1.27)$$

and

$$P_0 = \frac{1-\frac{p(1-s)}{s(1-p)}}{1-\left[\frac{p(1-s)}{s(1-p)}\right]^{N+1}}$$

$$P_0 = 1-\frac{p(1-s)}{s(1-p)} \qquad (1.28)$$

If someone has a $Geom/Geom/1/N$ queueing system, the transition probabilities are the same. with the exception that a $a_{0,0}$ = 1-p and $a_{0,1}$ = p. This means that a client arriving to an empty queue must wait at least until the following slot for service. In a manner similar to the foregoing, one may demonstrate that

$$P_n = \frac{p^n(1-s)^{n-1}}{s^n(1-p)^n}P_0, \qquad n = 1,2,....N \qquad (1.29)$$

and

$$P_0 = \frac{1-s}{\frac{1-\left[\frac{p(1-s)}{s(1-p)}\right]^{N+1}}{1-\frac{p(1-s)}{s(1-p)}}-s} \qquad (1.30)$$

This naturally becomes the case for a   queueing system with an unlimited buffer

$$P_0 = \frac{1-s}{\frac{1}{1-\frac{p(1-s)}{s(1-p)}}}-s$$

or

$$P_0 = 1-\frac{p}{s} \qquad (1.31)$$

Recall that packets entering and exiting the queue inside the same slot contribute to the average throughput of a queueing system including virtual cut-through when calculating the system's average throughput when there is a queue "empty."

A space division packet switch queue

An essential problem concerning phones and, later on, multiprocessor computers has been the way to link N inputs and N outputs. That is, how to build a box similar to the one in Figure 1.3 that simultaneously offers connecting paths from many inputs to multiple outputs. In a regular telephone conversation, the routes in the connectivity network are real circuits. Now that packet switching technology has advanced, bits go via the interconnection network in packets. A packet is a condensed collection of digital bits, some of which reflect the actual data being sent, while others of which are used to convey control information like the destination address.

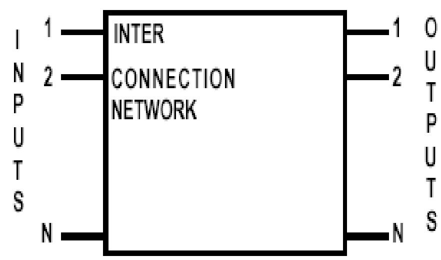

Fig. 1.3 : Interconnection network

Cross-bar architecture is an easy technique to construct an interconnection network. In a crossbar switch with N x N (Franklin,1981) , as figure 1.4 Say horizontal cables (buses) are used to link inputs, and vertical buses are used to connect outputs. At each intersection of a horizontal and vertical bus, a switch that may be independently closed or opened is included. The cross points of the connectivity are these. A closed switch, shown by a "X" in the diagram, makes a route from the input to the output. The term "space division" is applied to a crossbar switch, for instance, since the several paths that travel through it are divided spatially.

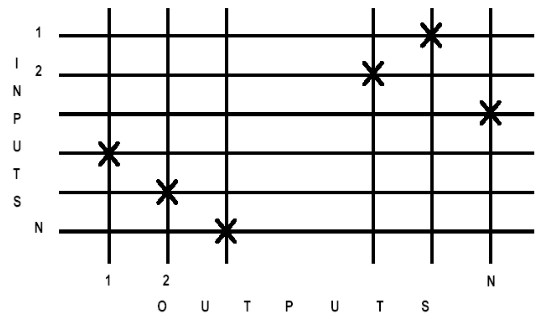

Fig. 1.4 NXN crossbar switch

To reduce the number of cross points, they result in blockage. Stated differently, not every input can have a simultaneous connection to every output. An input is deemed blocked when it is unable to reach an output. Due to the fact that blocking networks decrease the quantity of cross points a conventional indicator of switch complexity and because traffic patterns may require that few inputs be active at any given time, blocking networks can be helpful.

VI. CONCLUSION

Students, researchers, and engineers who want to understand the fundamentals of discrete-time queues should be able to read this work. All you need to know is the fundamentals of probability and stochastic processes. While the paper discusses important difficulties in discrete-time queues, it should also be helpful to those who are experienced Using systems that operate in continuous time and would prefer an approachable introduction to them. In conclusion, there are a wide variety of applications for queueing theory. In order to understand how daily social life develops, queuing theory is crucial. Every one of the applications covered above is useful, and real-world examples demonstrate their value.

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