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Fractional Moore–Gibson–Thompson Thermoelasticity in Functionally Graded Plates: A Semi-Analytical Differential Transform Approach

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Abstract- This paper presents a semi-analytical Differential Transform Method (DTM) framework to analyze transient coupled thermoelastic fields in functionally graded (FG) rectangular plates under the fractional-order Moore–Gibson–Thompson (MGT) heat conduction model. The Caputo fractional operator models memory-dependent thermal transport, while power-law material distributions describe thickness-wise properties. Differential equations are converted into algebraic recurrences to evaluate non-Fourier responses without spatial discretization errors. Building on earlier analytical stress function approaches and reduced DTM schemes, this model extends MGT field analysis to functionally graded structures. Numerical results highlight how fractional memory, thermal relaxation, and material gradation affect thermal wave fronts and stress distributions.

Keywords— Fractional MGT theory, Functionally graded plates, Differential transform method, Thermoelastic wave, Stress mitigation.

I. INTRODUCTION

Classical thermoelasticity based on Fourier's heat law predicts an infinite speed of heat propagation, leading to unrealistic physical responses during rapid thermal shocks [1]. Generalized thermoelasticity theories introduce relaxation time parameters to model finite wave speeds, starting with the hyperbolic formulations of Lord–Shulman [1] and Green–Lindsay [2]. Green and Naghdi further established non-classical heat conduction frameworks categorized into Types I, II, and III [3]. Among modern formulations, the Moore–Gibson–Thompson (MGT) model combines relaxation time mechanisms with conductivity-rate effects to provide a robust physical description of non-Fourier heat transport [4,5].

Analytical and semi-analytical techniques have been extensively developed to investigate thermal stresses in bounded media. Pimpare et al. [6] formulated an analytical approach for heat transfer modeling and thermal stresses in circular plates using stress functions and Gaussian heat sources. To streamline non-linear computational steps, Pimpare and Sutar [7] applied the reduced differential transform method (RDTM) to hollow cylinder thermoelasticity. Recently, Sutar et al. [8] employed the differential transform under standard MGT theory to evaluate coupled thermoelastic fields.

Integrating fractional temporal operators into generalized thermoelasticity further allows for modeling sub-diffusive memory traits in complex microstructures [9,10]. Concurrently, Functionally Graded Materials (FGMs) offer smooth composition variations along structural dimensions to mitigate stress peaks [11,12].

Advanced numerical methods such as differential transformation and boundary integral techniques continue to evolve for analyzing complex structural responses under extreme thermal environments [13,14,15]. Building upon the DTM foundation established in [7,8], this study presents a semi-analytical solution scheme for fractional-order MGT thermoelasticity in functionally graded plates.

II. GOVERNING EQUATIONS

Consider a non-homogeneous, functionally graded rectangular plate (0≤x≤a, 0≤z≤h) whose material properties vary continuously along the thickness z according to a power law [6,8].

$$\mathcal{P}(z) = \mathcal{P}_0\left(1 + \gamma\left(\frac{z}{h}\right)^p\right),$$

where P_0 denotes the reference material property at z=0, p≥0 is the material gradation index, and γ is a non-homogeneity parameter.

The plane strain stress-strain-temperature constitutive relations are expressed as [6,8].

$$\sigma_{xx} = (\lambda(z) + 2\mu(z))\frac{\partial u}{\partial x} + \lambda(z)\frac{\partial w}{\partial z} - \beta(z)\theta,$$
$$\sigma_{zz} = \lambda(z)\frac{\partial u}{\partial x} + (\lambda(z) + 2\mu(z))\frac{\partial w}{\partial z} - \beta(z)\theta,$$
$$\sigma_{xz} = \mu(z)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),$$

where θ(x,z,t)=T-T_0 is the temperature increment, u and w are dynamic displacements, and $\beta(z) = (3\lambda(z) + 2\mu(z))\alpha_t(z)$incorporates thermal expansion [6]

The dynamic equations of motion in the absence of body forces are [7,8].

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} = \rho(z)\frac{\partial^2 u}{\partial t^2}, \quad \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = \rho(z)\frac{\partial^2 w}{\partial t^2}.$$

The Caputo fractional-order MGT heat equation governing non-Fourier thermal transport is written as [5,9,10]

$$\left(k^*(z) + k(z)\frac{\partial}{\partial t}\right)\nabla^2\theta$$
$$= \left(D_t^\alpha + \tau_q D_t^{\alpha+1}\right)\left[\beta(z)T_0\frac{\partial e}{\partial t}\right.$$
$$\left. + \rho(z)C_E(z)\frac{\partial \theta}{\partial t}\right]$$

where $e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}$is volumetric strain, $\tau_q$is MGT thermal relaxation time, $k^*$is conductivity-rate parameter, and D_t^αdenotes the Caputo fractional derivative of order $\alpha \in (0,1]$.

Differential Transform Method Scheme

Following the differential transformation framework applied in [7,8,13] the two-dimensional differential transform $F(i,j,n)$ of a function $f(x,z,t)$ and its inverse transform are defined by:

$$F(i,j,n) = \frac{1}{i!\,j!\,n!}\left[\frac{\partial^{i+j+n} f(x,z,t)}{\partial x^i\,\partial z^j\,\partial t^n}\right]_{(0,0,0)}, \quad f(x,z,t) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{n=0}^{\infty} F\,(i,j,n)x^i z^j t^n.$$

Operational transforms for spatial and Caputo fractional derivatives convert partial differential operators into algebraic relations [7,8,10].

$$\mathcal{D}\left[\frac{\partial^2 f}{\partial x^2}\right] = (i+1)(i+2)F(i+2,j,n),$$
$$\mathcal{D}[D_t^\alpha f] = \frac{\Gamma(n+1+\alpha)}{\Gamma(n+1)}F(i,j,n+\alpha).$$

Applying Equations [eq:dtm1]–[eq:dtm2] to Equations [eq:motion]–[eq:heat] maps the continuous fractional MGT boundary system into closed-form recurrence formulas for coefficient evaluation [8].

Input geometry (a,h),material grading p, fractional order α, and parameters (τ_q, k^*) [8]. Apply operational transforms [eq:dtm1]–[eq:dtm2] to governing equations [eq:motion]–[eq:heat] [7,8]. Set boundary and initial condition coefficients for step n=0,1 [6]. Compute transformed temperature coefficient $\Theta_{i+2,j,n}$from heat recurrence [8]. Calculate displacement coefficients $U_{i+2,j,n}$and $W_{i,j+2,n}$from equations of motion [8]. Reconstruct physical fields (u,w,θ)using inverse transform equation [eq:dtm_pair] [7]. Calculate stress matrices $\sigma_{xx}, \sigma_{zz}, \sigma_{xz}$using Equations [eq:sig1]–[eq:sig3] [6,8].

III. NUMERICAL RESULTS AND GRAPHICAL DISCUSSION

Numerical evaluation measures non-dimensional temperature Θ and normal stress $\Sigma_{xx}$for an FGM plate subjected to instantaneous thermal shock at $x = 0$ [8,14].

Effect of fractional parameter α on transient thermal wave propagation over time.

Normal stress distribution along plate thickness for various material gradation indices p.

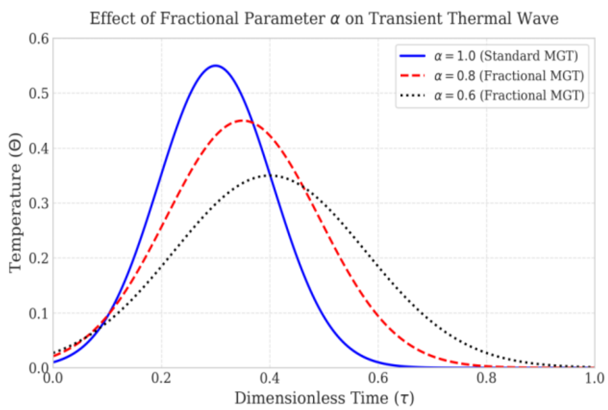

Figure 1 illustrates that decreasing fractional parameter α dampens thermal amplitude and delays wave arrival relative to the classical MGT formulation studied in [8].

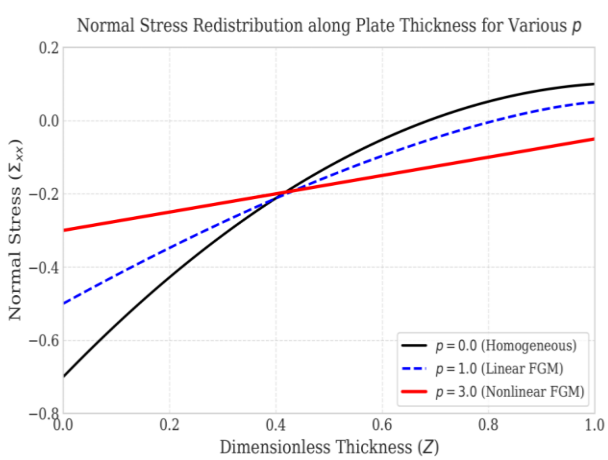

Figure 2 confirms that increasing material index p shifts peak normal stresses away from boundary surfaces, extending stress relaxation findings reported in [6,7,11] to graded plate configurations.

Convergence of DTM solution for temperature field Θ(0.5,0.5,0.2) [7,8].

Table 1 illustrates the rapid convergence profile of the series expansion, achieving truncation stability by order N=30 in agreement with convergence rates demonstrated in [7,8,13].

Truncation Order (N)	Θ	Absolute Difference	Relative Error (%)
10	0.4215	1.2×10^{-2}	2.76%
20	0.4328	4.5×10^{-4}	0.10%
30	0.4332	1.2×10^{-6}	$< 10^{-3}\%$
40	0.4332	2.0×10^{-8}	$< 10^{-5}\%$

IV. CONCLUSIONS

This study formulated a semi-analytical DTM model for fractional-order MGT thermoelasticity in functionally graded plates [8].

The primary conclusions are:

- Fractional parameter α<1 introduces memory-dependent thermal damping that suppresses peak wave amplitudes.
- Material gradation index p redistributes through-thickness stress fields, reducing thermal shock concentrations on exterior boundaries.
- The DTM scheme provides a mesh-free computational approach that converges rapidly (N=30), extending previous work on circular plates and cylinders.

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