

A Computationally Efficient Alternative to Mpc for Real-Time Torque Vectoring in 4wid Evs

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Abstract- Four-Wheel Independent Drive (4WID) electric vehicles offer unprecedented authority over individual wheel forces, enabling propulsion-based yaw stability control through torque vectoring. Model Predictive Control (MPC) has emerged as the dominant academic paradigm for multi-objective vehicle dynamic control owing to its constraint-handling capability and predictive horizon structure. However, the online solution of constrained quadratic programs incurs $O(n^3)$ computational complexity with respect to the prediction horizon length, rendering real-time deployment on standard automotive embedded controllers impractical for most production-grade applications. This paper proposes an architecture comprising a Sliding Mode Control (SMC) yaw stability controller integrated with a two-dimensional (2D) motor efficiency map (η -map) based torque allocator that resolves the corrective wheel torque distribution problem in $O(1)$ time via direct memory lookup. The SMC formulates a sliding surface combining yaw rate error and sideslip angle error, deploying a boundary-layer saturation function to eliminate chattering while preserving robust stability margins under nonlinear tire dynamics and variable road friction. The efficiency-aware torque allocator pre-computes optimal torque-speed operating regions offline and retrieves them via a 2D lookup table at each control step, achieving multi-objective performance simultaneous stability enhancement and energy efficiency improvement without requiring any online optimization. Comparative MATLAB/Simulink evaluation under a standardized step-steer maneuver at 60 km/h on a dry surface ($\mu = 0.85$) demonstrates that the proposed SMC reduces yaw rate root-mean-square error (RMSE) by 51.2% relative to the proportional (P) baseline, by 33.5% relative to PID, and by 37.6% relative to the Linear Quadratic Regulator (LQR), while consuming only 139.8 kJ compared to the highest-consumption controller at 142.7 kJ. Theoretical execution time analysis confirms that the proposed architecture is compatible with 1 ms embedded controller cycle times on standard 32-bit automotive-grade microcontrollers, a target that MPC-based alternatives cannot reliably meet without dedicated co-processor hardware.

Keywords: Keywords: Four-Wheel Independent Drive (4WID) Electric Vehicles, Sliding Mode Control (SMC), Yaw Stability Control, Torque Vectoring, Torque Allocation, Motor Efficiency Map (η -map), Sideslip Angle, Model Predictive Control (MPC), Real-Time Embedded Control, Energy Efficiency, MATLAB/Simulink, Nonlinear Vehicle Dynamics, Robust Control.

I. INTRODUCTION

1. The 4wid Electric Vehicle Control Opportunity

The global transition toward electrified mobility has fundamentally altered the design space of automotive control systems. Four-Wheel Independent Drive (4WID) electric vehicles, in which each wheel is powered by a dedicated in-wheel or near-wheel motor, represent the most architecturally radical manifestation of this transition [1]. By eliminating mechanical differentials and driveshafts, 4WID platforms enable fully decoupled

torque commands at each corner of the vehicle, affording a degree of actuation flexibility that is simply unattainable in internal combustion engine drive-trains [3]. This independent actuation enables torque vectoring the deliberate asymmetric distribution of drive torque across the left and right wheels to generate corrective yaw moments that stabilize the vehicle during cornering, emergency maneuvers, or adverse road friction conditions [5].

The near-instantaneous torque response of electric motors, with force-generating bandwidth several orders of magnitude higher than mechanical

differentials or brake-based Electronic Stability Control (ESC), makes 4WID vehicles ideal candidates for propulsion-based Direct Yaw Moment Control (DYC) [7]. Unlike braking-based correction, which dissipates kinetic energy and reduces vehicle speed, motor-based torque vectoring preserves momentum while providing high-bandwidth yaw regulation. However, the transition from one actuator (single ICE output) to four independently commandable actuators creates an over-actuated control problem in which the number of available control inputs exceeds the number of controlled degrees of freedom requiring a principled torque allocation strategy to resolve the redundancy [4].

Yaw rate and sideslip angle constitute the two primary state variables that torque vectoring systems must regulate [6]. Excessive sideslip indicates impending oversteer and loss of lateral stability, while yaw rate deviation from the desired bicycle-model reference signals understeer or yaw instability. The central control challenge is to generate precise corrective yaw moments that drive these states to their reference trajectories under conditions of nonlinear tire behavior, uncertain road friction, and rapidly time-varying lateral loads all within the hard real-time constraints imposed by automotive embedded controllers.

2. The Mpc Dominance And Its Computational Cost

Model Predictive Control has attracted sustained research interest as a framework for torque vectoring and vehicle dynamics control, particularly in academic settings [8]. Its appeal is well-founded: MPC can explicitly represent inequality constraints on wheel torque magnitudes, tire friction circle limits, and maximum permissible yaw rate and sideslip deviations; it minimizes a multi-step cost function that simultaneously penalizes yaw tracking error, sideslip excursion, and control effort; and it can integrate energy efficiency terms directly into the quadratic objective [20]. These properties make MPC appear to be an ideal solution for the multi-objective 4WID torque vectoring problem.

In practice, however, standard MPC requires the online solution of a constrained Quadratic Program (QP) at every control step. For a linear time-invariant prediction model of dimension n_x with prediction horizon N_p

and control horizon N_c , the QP has $N_c \cdot n_u$ optimization variables and $2N_p \cdot n_x$ constraint inequalities, where n_u is the number of control inputs. The computational cost of solving this QP using interior-point or active-set methods scales as $O((N_c \cdot n_u)^3)$ per iteration equivalently $O(n^3)$ in the combined problem dimension [8]. For a 4WID vehicle with four motor torque inputs ($n_u = 4$), state vector of dimension $n_x = 6$ (lateral velocity, yaw rate, sideslip, and three additional coupled states), and a modest 10-step prediction horizon ($N_c = 10$), the QP dimension is 40, yielding approximately 64,000 floating-point operations per QP iteration, with typically 10–50 iterations required for convergence. At a 1 kHz control loop, this demands 640,000 to 3,200,000 floating-point operations per millisecond a computation rate that exceeds the capability of most production-grade 32-bit automotive microcontrollers such as the Infineon TriCore AURIX family or the Renesas RH850 series, which sustain approximately 200–400 MFLOPS under realistic embedded workloads [8].

The computational burden of MPC also introduces unpredictable worst-case execution time (WCET), which violates the determinism requirement of ISO 26262-compliant automotive systems. Real-time MPC implementations in the literature either reduce horizon length to $N_p = 3-5$ (sacrificing predictive optimality), require a dedicated floating-point co-processor or FPGA accelerator (increasing hardware cost and complexity), or operate at reduced sampling rates of 50–200 Hz (insufficient for the bandwidth requirements of high-speed torque vectoring) [8], [20]. These constraints create a fundamental gap between MPC's theoretical attractiveness and its practical deployability on production ECU hardware.

3. The Industrial Feasibility Gap

The divergence between academic control theory and industrial embedded reality is a well-recognized challenge in automotive control engineering. Academic MPC publications frequently report impressive simulation performance but are evaluated using floating-point desktop MATLAB solvers such as OSQP, qpOASES, or quadprog at unlimited computational budgets, without accounting for fixed-point arithmetic, memory constraints, or interrupt-driven execution environments [8]. Automotive ECUs, by contrast,

typically operate under constraints of 256–512 kB of RAM, 32-bit integer or single-precision floating-point arithmetic, deterministic execution within 0.5–2.0 ms interrupt service routines, and strict ISO 26262 requirements for WCET certification.

Industrial torque vectoring implementations therefore predominantly rely on computationally simpler control structures gain-scheduled PID controllers, rule-based torque maps, or LQR regulators with pre-computed feedback gains that offer deterministic $O(1)$ or $O(n)$ execution time [9], [10]. These approaches sacrifice the multi-objective optimality of MPC but achieve the latency, determinism, and memory footprint required by production ECUs. The result is a persistent gap: industry employs controllers that are tractable but theoretically suboptimal, while academia proposes controllers that are theoretically rich but practically infeasible.

4. Proposed Contribution

This paper proposes and evaluates an architecture that bridges this gap by combining a robust nonlinear Sliding Mode Controller with an $O(1)$ 2D lookup-table torque allocator to deliver MPC-level multi-objective performance within embedded ECU constraints. The specific contributions of this work are fourfold:

A nonlinear SMC yaw stability controller formulated on a 2DOF lateral-yaw vehicle model is designed, which achieves finite-time convergence of yaw rate error and sideslip angle error to a sliding surface, providing robustness against nonlinear tire dynamics, parameter uncertainty, and road friction variation properties that linear controllers (PID, LQR) cannot guarantee [2], [7].

An efficiency-aware torque allocation algorithm based on pre-computed 2D motor η -maps that resolves the over-actuated 4WID torque distribution problem in $O(1)$ time, simultaneously satisfying the yaw moment demand from the SMC and biasing torque toward high-efficiency operating regions, thereby achieving the dual-objective performance of MPC without any online optimization [1], [5]. A rigorous computational complexity analysis comparing the proposed architecture against standard MPC, providing theoretical execution time estimates and simulated ECU processor load that quantify the feasibility advantage of

the proposed method for production automotive deployment.

Simulation validation of the proposed SMC + η -map architecture against three linear baseline controllers (P, PID, LQR) under a standardized step-steer maneuver, demonstrating quantitative improvements in yaw tracking RMSE, sideslip stabilization, and energy efficiency.

II. VEHICLE DYNAMICS AND CONTROL ARCHITECTURE

1. Nonlinear 2dof Lateral-Yaw Vehicle Model

The control-oriented plant model is the Two-Degree-of-Freedom (2DOF) bicycle model, extended to incorporate independent wheel torques from the 4WID architecture. The two state variables are the lateral velocity v_y (m/s) and the yaw rate r (rad/s). The governing differential equations are [36], [39]:

$$\begin{aligned} m(\dot{v}_y + v_x \cdot r) &= F_{yf} + F_{yr} \\ I_z \cdot \dot{r} &= l_f \cdot F_{yf} - l_r \cdot F_{yr} + M_{TV} \end{aligned}$$

where $m = 1500$ kg is the vehicle mass, $I_z = 2800$ kg·m² is the yaw moment of inertia, v_x is the longitudinal velocity (treated as a slowly varying external parameter), $l_f = 1.2$ m and $l_r = 1.6$ m are the distances from the center of gravity (CG) to the front and rear axles, respectively, F_{yf} and F_{yr} are the resultant lateral tire forces at the front and rear axles, and M_{TV} is the torque-vectoring yaw moment generated by the differential longitudinal forces from the four independent motors.

The tire slip angles at the front and rear axles are computed using nonlinear trigonometric expressions to improve accuracy over the small-angle linearization [37]

$$\begin{aligned} \alpha_f &= \delta - \arctan((v_y + l_f \cdot r) / v_x) \\ \alpha_r &= -\arctan((v_y - l_r \cdot r) / v_x) \end{aligned}$$

where δ is the front wheel steering angle. Lateral forces are modeled using linear cornering stiffness

$$\begin{aligned} C_f &= C_r = 80,000 \text{ N/rad per axle:} \\ F_{yf} &= C_f \cdot \alpha_f, \quad F_{yr} = C_r \cdot \alpha_r \end{aligned}$$

Longitudinal forces at each wheel $i \in \{FL, FR, RL, RR\}$ are derived from the individual motor torque commands T_i and wheel radius $R_w = 0.32$ m:

$$F_{xi} = T_i / R_w, \text{ subject to } |F_{xi}| \leq \mu \cdot F_{zi}$$

where F_{zi} is the normal load at wheel i and μ is the road friction coefficient. The torque-vectoring yaw moment from longitudinal force asymmetry between the left and right wheels is:

$$M_{TV} = (t_f/2)(F_{xFR} - F_{xFL}) + (t_r/2)(F_{xRR} - F_{xRL})$$

where $t_f = t_r = 1.55$ m are the front and rear track widths. This model is implemented in a MATLAB Function block in Simulink, providing a fully transparent nonlinear simulation plant for controller evaluation [38].

A. B. SMC Yaw Stability Controller Design

III. REFERENCE MODEL AND ERROR STATES

The desired yaw rate reference r_{des} is derived from a linear steady-state bicycle model as [44], [45]:

$$r_{des} = v_x \cdot \delta / (L_f + L_r) \cdot 1 / (1 + K_{us} \cdot v_x^2)$$

where K_{us} is the understeer gradient, computed from front and rear axle stiffness and mass distribution. The desired sideslip angle β_{des} is set to zero (or a small bounded reference derived from the reference bicycle model lateral velocity). The yaw rate error and sideslip error are:

$$e_r = r_{meas} - r_{des}, \quad e_\beta = \beta_{est} - \beta_{des}$$

IV. SLIDING SURFACE DESIGN

The sliding surface combines the two error states with weighting coefficients c_1 and c_2 , selected to trade off yaw tracking versus sideslip suppression [2], [46]:

$$s = c_1 \cdot e_r + c_2 \cdot e_\beta$$

When $s = 0$, the system operates on the sliding manifold, and the error dynamics reduce to a stable first-order system. For the proposed implementation, $c_1 = 1.0$ and $c_2 = 0.5$, reflecting the higher priority assigned to yaw rate tracking.

V. CONTROL LAW AND CHATTERING MITIGATION

The corrective yaw moment M_{yaw} commanded by the SMC is [7], [50]:

$$M_{yaw} = -K \cdot \text{sat}(s / \varphi)$$

where $K = 2500$ N·m is the switching gain, $\varphi = 0.05$ rad/s is the boundary-layer thickness, and $\text{sat}(\cdot)$ denotes the saturation function:

$$\text{sat}(x) = x \text{ if } |x| \leq 1, \quad \text{sign}(x) \text{ if } |x| > 1$$

The boundary-layer saturation function replaces the discontinuous sign function of classical SMC, eliminating high-frequency chattering that would otherwise excite structural resonances and cause unacceptable torque ripple in the motors. Within the boundary layer ($|s| \leq \varphi$), the controller transitions continuously to a proportional law, providing smooth yaw moment commands suitable for production actuation systems [2], [7].

Lyapunov stability analysis confirms that the control law drives s to zero in finite time when $|s| > \varphi$, and constrains the trajectory within the boundary layer for all subsequent time, guaranteeing bounded yaw rate error [44].

VI. SIDESLIP ESTIMATION

Since sideslip angle cannot be directly measured by standard production sensors, it is estimated kinematically from the lateral and longitudinal velocity states available in simulation [29]:

$$\beta_{est} = \arctan(v_y / v_x)$$

In a production implementation, this would be replaced by an Extended Kalman Filter (EKF) or a nonlinear observer fusing yaw rate, lateral acceleration, and wheel speed measurements. The inherent robustness of SMC to estimation errors a fundamental property of sliding mode controllers once the sliding surface is reached means that moderate sideslip estimation inaccuracies do not destabilize the control loop [2], [29].

VII. EFFICIENCY-AWARE TORQUE ALLOCATION: THE O(1) SOLUTION

1. Problem Formulation And Mpc Comparison

The torque allocation problem is to determine the four individual motor torque commands T_{FL} , T_{FR} , T_{RL} , T_{RR} that simultaneously satisfy (i) the driver's total torque demand T_{req} , (ii) the corrective yaw moment M_{yaw} commanded by the SMC, (iii) the friction capability constraints at each wheel, and (iv) an energy efficiency objective that minimizes aggregate motor power loss. This is precisely the class of multi-objective constrained optimization problem for which MPC is typically proposed in the literature [4], [10].

A standard MPC torque allocator formulates this as a Quadratic Program (QP) at each control step k :

$$\begin{aligned} \min \quad & \sum_i [w_1(T_i - T_{i,opt})^2 + w_2 \cdot P_{loss,i}(T_i, \omega_i)] \\ \text{subject to: } & \sum_i T_i = T_{req}, \quad \sum_i F_{xi} \cdot d_i = M_{yaw}, \quad |T_i| \\ & \leq T_{max}, \quad |F_{xi}| \leq \mu \cdot F_{zi} \end{aligned}$$

where d_i is the lateral moment arm of wheel i ($\pm t/2$). This QP has four optimization variables, four to six inequality constraints, and two equality constraints. Solving it at 1 kHz requires an embedded QP solver capable of completing each solve within 1 ms a requirement that standard automotive microcontrollers cannot reliably satisfy without co-processor acceleration [8], [20].

2. The 2d Lookup Table Architecture

The proposed architecture decouples the optimization from the control loop by pre-computing the efficiency-optimal torque allocation strategy offline and encoding it as a 2D lookup table indexed by the motor angular speed ω and the required torque T . This is conceptually analogous to the approach used for optimal motor drive point selection in hybrid vehicle energy management, where offline dynamic programming produces a policy table that is subsequently queried at O(1) cost online [1], [5].

The motor efficiency map $\eta(\omega, T)$ is defined over a grid:

$$T_{grid} \in [-200, 200] \text{ Nm (step: 20 Nm)}, \quad \omega_{grid} \in [0, 5000] \text{ rpm (step: 100 rpm)}$$

Efficiency values $\eta(\omega, T) \in [0.70, 0.95]$ are stored in the η_{map} matrix, representing the ratio of mechanical output power to electrical input power at each (ω, T) operating point. The electrical input power and power loss at wheel i are:

$$P_{elec,i} = T_i \cdot \omega_i / \eta(\omega_i, T_i), \quad P_{loss,i} = P_{elec,i} - T_i \cdot \omega_i$$

C. O(1) Allocation Algorithm

At each control step, the torque allocator executes the following deterministic fixed-time procedure [3], [53]:

Step 1 Compute torque differential: The corrective left-right torque differential required to generate M_{yaw} is:

$$\Delta T = M_{yaw} \cdot R_w / t$$

Step 2 Distribute base torque: The driver's total torque demand is split equally across all four wheels as a base command:

$$T_{base} = T_{req} / 4$$

Step 3 Apply yaw correction and η -map adjustment: For each wheel, the provisional torque command is:

$$T_{FL} = T_{RL} = T_{base} - \Delta T/2, \quad T_{FR} = T_{RR} = T_{base} + \Delta T/2$$

An efficiency correction is then applied: for each wheel i , if $\eta(\omega_i, T_i) < \eta_{threshold}$ (set to 0.80 in this implementation), the torque command is scaled toward the peak-efficiency torque at the current motor speed, retrieved in O(1) time via bilinear interpolation of the pre-loaded η_{map} table.

Step 4 Enforce constraints: Each torque command is clipped to the friction-limited maximum:

$$T_i = \text{clip}(T_i, -\mu \cdot F_{zi} \cdot R_w, +\mu \cdot F_{zi} \cdot R_w)$$

This four-step procedure involves only algebraic operations, a single table lookup with bilinear interpolation (four multiplications and two additions), and four saturation operations. Its Worst-Case Execution Time (WCET) is fully bounded and deterministic a critical property for ISO 26262 ASIL-D compliance. The entire procedure executes in O(1) time regardless of the desired prediction horizon, number of constraints, or optimization tolerance, in stark contrast to the $O(n^3)$ scaling of MPC QP solvers [8].

VIII. COMPUTATIONAL ANALYSIS AND REAL-TIME FEASIBILITY

1. Theoretical Complexity Comparison

Table I: Theoretical Computational Complexity Comparison

Parameter	MPC (N _p =10, N _c =10)	MPC (N _p =5, N _c =5)	SMC + η - map (Proposed)
QP / Opt. Variables	40	20	N/A
Complexity Order	$O(n^3) \approx$ $O(64,000)$	$O(n^3) \approx$ $O(8,000)$	$O(1) \approx$ $O(30)$
FLOP Estimate (per step)	640k–3.2M	80k– 400k	~30
WCET @ 32- bit ECU	Unbounded / >5 ms	~2–10 ms	<0.05 ms
Feasible @ 1 kHz	No (without co- processor)	Marginal	Yes ($\times 20$ margin)
Deterministic WCET	No	No	Yes
RAM Requirement	~50–200 kB	~15–50 kB	~12 kB (η -map)

2. Ecu Processor Load Estimation

To quantify the practical processor load difference between the proposed method and MPC, a theoretical benchmarking analysis is conducted based on the computational profile of a representative 32-bit automotive ECU specifically, a single-core 200 MHz processor with sustained throughput of approximately 200 MFLOPS under realistic load, representative of the Infineon TriCore TC27x or Renesas RH850/P1H-C families widely used in chassis control applications [8], [10]. For the proposed SMC + η -map architecture, the computational budget per 1 ms control cycle is itemized as follows: SMC sliding surface evaluation requires 4 multiply-accumulate (MAC) operations and 2 comparisons (≈ 12 FLOPS); the saturation function computation requires 2 comparisons and 1 division (≈ 5 FLOPS); the torque allocator requires 6 additions, 4 multiplications, and 4 saturation operations (≈ 18 FLOPS); and the bilinear η -map interpolation requires 8 multiplications and 6 additions (≈ 14 FLOPS). The total FLOP count per control step is approximately 49 FLOPS,

yielding a processor utilization of $49 / (200 \times 10^6) \approx 0.024\%$ of the available 200 MHz capacity. Even accounting for context switching, interrupt overhead, and code branching, the estimated real-world processor load is below 1% leaving more than 99% of the ECU computational budget available for other vehicle functions (diagnostics, communication, ADAS processing). For a standard 10-step MPC implementation using the OSQP solver (a state-of-the-art embedded QP solver), the estimated computation per step is 320,000–1,600,000 FLOPS depending on problem warm-starting and iteration count. At 200 MFLOPS sustained throughput, this corresponds to 1.6–8.0 ms per control step exceeding the 1 ms cycle time budget by a factor of 1.6 to 8. A 5-step MPC reduces this to approximately 0.4–2.0 ms, which is marginal and non-deterministic. The WCET cannot be certified under ISO 26262 because QP solver iteration count depends on the problem data at each step, rendering the execution time distribution unbounded in the worst case.

Table II: Simulated ECU Processor Load at 1 kHz Control Rate (200 MFLOPS Reference ECU)

Metric	MPC (N _p =10)	MPC (N _p =5)	SMC + η - map	Reduction vs MPC- 10
FLOP/step (est.)	640k–3.2M	80k– 400k	~49	$\times 13,000$ – 65,000
Execution time (est.)	1.6–8.0 ms	0.4–2.0 ms	<0.05 ms	$\times 32$ –160
CPU Load (%)	160–800%*	40– 200%*	<1%	N/A
Remaining budget	N/A (overloaded)	Marginal	>99%	
RAM usage (typical)	50–200 kB	15–50 kB	~12 kB	$\times 4$ –17
WCET certified	No	No	Yes	

* Values > 100% indicate the method cannot meet 1 ms cycle time without co-processor hardware.

3. Memory Footprint Analysis

The η _map lookup table for a single motor, sampled over a 21×51 grid of torque-speed combinations (T_{grid} : 21 breakpoints, ω_{grid} : 51 breakpoints), requires $21 \times 51 \times 4$ bytes (single-precision float) = 4,284 bytes $\approx$ 4.2 kB per motor. For four motors with shared efficiency characteristic, the total η _map memory footprint is approximately 4.2 kB well within the 256–512 kB RAM budget of automotive microcontrollers. The T_{grid} and ω_{grid} arrays require an additional 284 bytes, bringing the total data memory requirement of the allocator to approximately 4.5 kB [1], [5].

By comparison, an MPC implementation with prediction horizon $N_p = 10$, control horizon $N_c = 10$, and state dimension $n_x = 6$ must store the Hessian matrix ($40 \times 40 = 1600$ floats = 6.4 kB), the gradient vector (40 floats), the constraint matrix ($160 \times 40 = 6400$ floats = 25.6 kB), and warm-start state vectors (80 floats), totaling approximately 32.5 kB for the QP data structures alone excluding the solver library code, which typically requires an additional 50–150 kB of code memory.

4. Relationship To The Mpc Cost Function

It is important to acknowledge that while the proposed $O(1)$ method achieves multi-objective performance comparable to MPC in the simulation results reported here, it does not solve the same mathematical problem as MPC. MPC minimizes a multi-step predictive cost over a finite horizon with explicit constraint handling, whereas the SMC + η -map architecture achieves stability through a Lyapunov-based sliding mode argument and achieves efficiency improvement through greedy single-step η -map bias. The proposed method therefore represents a computationally tractable approximation that achieves MPC-level outcomes in the operating regimes tested, rather than a formal equivalence.

The practical implication is that scenarios with tightly coupled multi-step constraint interactions such as cornering on a rapidly changing friction profile may benefit from MPC's predictive capability, but at a computational cost that the proposed architecture avoids entirely [8], [20].

IX. SIMULATION RESULTS

1. Simulation Setup

All simulations are executed in MATLAB/Simulink R2024b using the ode45 variable-step solver with a maximum step size of 0.001 s (1 kHz equivalent sampling). The vehicle dynamics block is implemented as a MATLAB Function block containing the full nonlinear 2DOF equations. Four controllers are evaluated under identical conditions: the Proportional (P) baseline, PID, LQR (with $Q = \text{diag}([100, 10])$ and $R = 0.001$), and the proposed SMC + η -map architecture. A standardized step-steer maneuver is applied: front wheel steering angle stepped to $\delta = 0.1$ rad at $t = 2$ s, held constant for the remaining 8 s of a 10 s simulation window. Vehicle speed is held constant at $v_x = 60$ km/h, road friction coefficient $\mu = 0.85$ (dry asphalt) [29], [37]. Vehicle and motor parameters are summarized in Table III.

Table III: Vehicle and Motor Parameters

Symbol	Description	Value
m	Vehicle mass	1500 kg
I_z	Yaw moment of inertia	2800 kg·m ²
l_f / l_r	CG to front / rear axle	1.2 m / 1.6 m
$t_f = t_r$	Track width	1.55 m
R_w	Wheel radius	0.32 m
$C_f = C_r$	Cornering stiffness (per axle)	80,000 N/rad
μ	Road friction coefficient	0.85 (dry)
v_x	Longitudinal speed	60 km/h
δ_{step}	Steering step magnitude	0.1 rad
$\eta_{\text{map range}}$	Motor efficiency range	0.70–0.95
SMC gains	c_1, c_2, K, φ	1.0, 0.5, 2500, 0.05

2. Yaw Rate Tracking Performance

Fig. 11 in the underlying simulation study presents the desired and actual yaw rate time histories for all four

controllers. Following the steering step at $t = 2$ s, the P controller exhibits the largest steady-state error, failing to generate sufficient corrective yaw moment under nonlinear tire loading and retaining a persistent tracking deviation throughout the maneuver.

The PID controller reduces steady-state error through integral action but introduces oscillatory transients immediately following the step, arising from derivative gain sensitivity to the rapid yaw rate gradient. The LQR controller produces a smoother transient than PID but exhibits a slow settling behavior as the tire dynamics exceed the validity range of its linearized gain matrix, causing under-compensation in the nonlinear slip-angle regime. The proposed SMC controller achieves the fastest convergence, with the actual yaw rate reaching and tracking the reference within approximately 0.4 s of the steering step and maintaining negligible tracking error throughout the steady-state phase, reflecting the sliding mode's ability to reject the nonlinear tire force disturbance and maintain the trajectory on the sliding surface [2], [44].

Table IV: Yaw Rate Tracking RMSE Comparison (Step-Steer Maneuver, $v_x = 60$ km/h, $\mu = 0.85$)

Controller	RMSE (rad/s)	Reduction vs. P (%)	Tracking Assessment
P (Baseline)	1.215		Poor
PID	0.892	26.6%	Good, oscillatory
LQR	0.950	21.8%	Fair, slow settling
SMC + η -map (Proposed)	0.593	51.2%	Excellent

The SMC achieves a 51.2% RMSE reduction relative to the P baseline, 33.5% reduction relative to PID, and 37.6% reduction relative to LQR. These improvements are attributable to the SMC's inherent nonlinear robustness: the sliding mode dynamics force the yaw rate error toward zero even when tire cornering stiffness deviates from its nominal value due to lateral load redistribution following the steer input [45], [50].

3. Sideslip Angle Analysis

Sideslip stability analysis evaluates the ability of each controller to suppress lateral body drift following the steering transient. The P controller allows the highest peak sideslip angle ($|\beta|_{\max} = 0.152$ rad) with a slow decay settling time of approximately 3.5 s, indicating insufficient stabilization authority under the increased lateral tire force demands. The PID controller reduces peak sideslip but introduces oscillatory recovery behavior due to integral windup during the high-slip transient. The LQR controller provides smoother decay but still reaches 0.151 rad peak sideslip, comparable to the P baseline, as it cannot fully compensate for the nonlinear tire behavior outside its linearization operating point.

Table V: Sideslip Angle Performance Summary

Controller	Peak $ \beta $ (rad)	Settling Time (s)	Stability Assessment
P (Baseline)	0.152	3.5	Poor
PID	0.150	2.8	Moderate, oscillatory
LQR	0.151	3.2	Fair
SMC + η -map (Proposed)	0.148	2.2	Excellent

The SMC achieves the lowest peak sideslip angle (0.148 rad) and the fastest settling time (2.2 s), despite the small absolute numerical differences between controllers. The dynamic significance of the SMC advantage lies in the oscillation-free recovery profile: the sliding mode enforces a monotonic return of the sideslip state to within the boundary layer, whereas PID exhibits repeated overshoot cycles that imply alternating understeer and oversteer corrections a handling characteristic that would be perceptible to the driver and indicative of reduced lateral stability margin [6], [33].

4. Energy Efficiency And Torque Allocation

Energy analysis quantifies the total electrical energy consumed by all four motors over the 10 s simulation window. The P controller records the lowest energy consumption at 138.5 kJ, which is attributable not to efficiency but to insufficient torque actuation it generates less corrective yaw moment and therefore draws less motor power, at the cost of poor stability

performance. The PID controller consumes the most energy at 142.7 kJ, driven by its oscillatory yaw moment commands that repeatedly actuate the motors through high-amplitude cycles, increasing both average power draw and instantaneous loss events in low-efficiency motor operating regions.

Table VI: Electrical Energy Consumption Summary (10 s Step-Steer Maneuver)

Controller	Energy (kJ)	vs. PID (Highest)	Stability + Efficiency
P (Baseline)	138.5	-3.0%	Low energy, unsafe
PID	142.7	Reference	Oscillatory, inefficient
LQR	141.5	-0.8%	Moderate
SMC + η -map (Proposed)	139.8	-2.0%	Optimal trade-off

The proposed SMC + η -map architecture consumes 139.8 kJ 2.0% less than PID and 1.2% less than LQR while delivering far superior stability performance.

This result validates the dual-objective design philosophy: the η -map efficiency correction biases torque toward higher-efficiency motor operating regions, reducing power losses without requiring online optimization.

Critically, the SMC's non-oscillatory yaw moment profile eliminates the current ripple penalty that inflates the PID energy figure, demonstrating that the smooth boundary-layer saturation function provides both a chattering mitigation and an energy efficiency benefit simultaneously [1], [5], [7].

5. Consolidated Performance Comparison

Table VII: Consolidated Multi-Metric Performance Comparison

Metric	P	PID	LQR	SMC+ η -map
Yaw Tracking (RMSE)	Poor	Good	Fair	Excellent
Sideslip Stability	Poor	Moderate	Fair	Excellent
Control Effort Quality	Under-actuation	Oscillatory	Smooth/limited	Decisive+stable
Energy Efficiency	Best (unsafe)	Poor	Fair	Good
Nonlinear Robustness	Poor	Fair	Poor	Excellent
Real-Time Feasibility	Yes	Yes	Yes	Yes (O(1))
ECU Processor Load	<1%	<1%	<1%	<1%
Overall Rank	4th	2nd-3rd	2nd-3rd	1st

X. DISCUSSION

1. Bridging Mpc-Level Performance And Ecu Constraints

The central thesis of this paper is that the proposed SMC + η -map architecture achieves multi-objective performance simultaneous yaw stability optimization and energy efficiency improvement that is functionally comparable to MPC, while imposing a computational burden that is four to five orders of magnitude lower. The simulation results presented in Section V substantiate this claim: the SMC + η -map method outperforms all three linear baselines in yaw tracking

accuracy, sideslip suppression, and energy consumption, categories that MPC optimizes explicitly through its quadratic cost function.

The mechanism by which this apparent equivalence is achieved deserves careful examination. MPC minimizes a multi-step predictive cost by solving a constrained QP, thereby simultaneously optimizing the control trajectory over a finite horizon. The SMC, by contrast, stabilizes the system through a Lyapunov-based switching control law that drives the error state to the sliding surface in finite time, while the η -map allocator achieves energy efficiency through a pre-computed

greedy selection policy rather than joint optimization. The two approaches solve related but not identical problems: MPC solves a globally constrained optimization over a horizon, while the proposed method solves a local stabilization problem (SMC) combined with a single-step efficiency maximization (η -map). In scenarios where multi-step constraint coupling is critical for example, a series of rapid successive steering inputs near tire force limits MPC's predictive capability may offer performance advantages. However, for the canonical step-steer and steady-state cornering scenarios that define the primary design envelope for passenger vehicle torque vectoring, the proposed architecture achieves equivalent or superior outcomes at a fraction of the computational cost [4], [8], [20].

2. Robustness As A Substitute For Prediction

A key insight from this work is that SMC's inherent robustness to model uncertainty serves as a functional substitute for MPC's explicit uncertainty handling through constraint margins and conservative horizon rollout. SMC does not require an accurate prediction model: once the sliding surface is reached, the closed-loop dynamics on the surface are model-independent, exhibiting the well-known invariance property of sliding mode control [2], [44]. This means that variations in tire cornering stiffness, vehicle mass loading, or road friction coefficient factors that can corrupt an MPC prediction model and require real-time parameter identification do not degrade SMC performance beyond the boundary-layer tolerance. This robustness property is particularly valuable for automotive applications, where model uncertainty is endemic and expensive real-time parameter estimation is often not feasible within ECU resource constraints [6], [33].

C. The $O(1)$ Efficiency Allocator as a Practical MPC Surrogate

The η -map lookup allocator achieves the energy efficiency objective of MPC's quadratic cost term through a different mechanism: rather than optimizing over all possible torque distributions at each step, it applies a pre-computed efficiency-maximizing rule derived from the known motor efficiency map. This offline-online decomposition a well-established technique in optimal control and operations research trades global optimality for computational tractability. The resulting policy is not globally optimal in the MPC

sense but achieves near-optimal performance in the energy consumption metric (139.8 kJ vs. 138.5 kJ for the unconstrained P baseline that sacrifices stability), validating the practical effectiveness of the approach [1], [5], [53].

The pre-computation of the η -map from motor characterization data is a one-time offline cost that can be absorbed into the ECU calibration process using standard automotive engineering tools (MATLAB Motor Control Blockset, INCA, or equivalent). Once stored in ECU flash memory, the map requires no updating during vehicle operation, provided the motor efficiency characteristics remain stable a reasonable assumption for the life of the motor under normal thermal management conditions [30].

3. Limitations And Scope

Several limitations of this study should be acknowledged explicitly. First, the 2DOF vehicle model does not capture roll dynamics, load transfer, or combined longitudinal-lateral tire slip, which are relevant for high-lateral-acceleration maneuvers ($>0.5g$). The proposed SMC's robustness will partially compensate for these modeling deficiencies, but validation against a high-fidelity 7DOF or CarSim model is a necessary next step before hardware deployment. Second, the sideslip estimator used in this study assumes availability of lateral velocity a quantity that is not directly measured in production vehicles and must be estimated by an EKF or observer. The integration of a production-grade sideslip observer would be required for hardware implementation. Third, the energy consumption comparison is conducted over a single 10 s maneuver; a more comprehensive energy cycle study over real-world drive cycle profiles would be necessary to quantify range improvement claims for a production vehicle.

XI. CONCLUSION

This paper has presented a computationally efficient alternative to Model Predictive Control for real-time torque vectoring in Four-Wheel Independent Drive electric vehicles, comprising a Sliding Mode Control yaw stability regulator and an $O(1)$ 2D motor efficiency map-based torque allocator. The central contribution is a rigorous computational complexity analysis

demonstrating that the proposed architecture achieves multi-objective performance simultaneous yaw stability optimization and energy efficiency improvement at an execution cost of approximately 49 FLOPS per control step, compared to 640,000–3,200,000 FLOPS for a standard 10-step MPC QP, a reduction of four to five orders of magnitude.

Simulation validation under a standardized step-steer maneuver confirms that the proposed SMC + η -map method reduces yaw rate RMSE by 51.2% relative to the P baseline, 33.5% relative to PID, and 37.6% relative to LQR, while consuming only 139.8 kJ compared to the highest-consumption controller at 142.7 kJ. Theoretical execution time analysis demonstrates compatibility with 1 ms embedded controller cycle times on standard 32-bit automotive-grade microcontrollers with less than 1% processor utilization, enabling more than 99% of the ECU budget to remain available for other vehicle functions.

The practical implication is that the proposed architecture enables production-grade deployment of robust, multi-objective torque vectoring on the standard ECU hardware already present in contemporary electric vehicles, without the need for dedicated co-processor acceleration or reduced-horizon approximations that compromise MPC's optimality.

This bridges the persistent gap between academic MPC theory and industrial embedded practice, providing a control solution that is simultaneously theoretically principled, computationally tractable, and industrially deployable.

Future work will extend the analysis to a 7DOF vehicle model and CarSim co-simulation environment, integrate an EKF-based sideslip observer for production-representative state estimation, and implement the architecture on a rapid prototyping ECU (dSPACE MicroAutoBox III) for hardware-in-the-loop validation. Coupling the proposed base architecture with a higher-level MPC path planner using SMC + η -map as the inner-loop stabilizer and MPC as an outer-loop reference governor operating at a reduced 10–50 Hz rate represents a particularly promising direction for high-performance autonomous vehicle applications

where the latency advantages of the proposed method and the predictive advantages of MPC can be combined without conflict.

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