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Dynamiflow AI An Explainable Mathematical Framework for Non Linear Differential Equations In Chatoic and Dynamical Systems Analysis

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Abstract- The analysis of nonlinear differential equations describing chaotic dynamical systems continues to be a challenging problem at the nexus of mathematical theory and practical computation. Most previous data-driven discovery approaches have focused on sacrificing either interpretability for flexibility or making assumptions about which terms appear in the governing equations. In this work, we present DynamiFlow AI, a mathematically interpretable framework that combines multi-scale Koopman neural operators with differentiable physics constraints and sparse symbolic distillation. Our method learns latent linear representations of the nonlinear dynamics as well as explicit governing equations from a differentiable candidate library, while ensuring consistency in the identified equations when considered from multiple augmented views of the trajectories. DynamiFlow AI is applied to benchmark problems in chaotic dynamical systems including the Lorenz-63 system, the Kuramoto-Sivashinsky equation, and a reaction-diffusion system. We show that our framework discovers correct structural information in the governing equations and retains accurate long-horizon trajectory behavior, while simultaneously providing directly interpretable symbolic coefficients of chaotic flows.

Key Word: Chaotic Dynamics, Koopman Operator, Explainable AI, Nonlinear Differential Equations, Symbolic Regression, System Identification

I. INTRODUCTION

The study of chaotic and complex dynamical systems through the analysis of nonlinear differential equations is essential for physics, engineering, and biology. In systems ranging from the Lorenz attractor to the spatiotemporal chaos of the Kuramoto-Sivashinsky equation, the ability to characterize and understand such behavior from data is fundamentally important [1][2]. However, the governing equations of chaotic dynamical systems are usually not known, and when they are known, they tend to be highly nonlinear and sensitive to initial conditions. As a result, classical techniques become brittle as the dimensionality increases [2][3].

The introduction of data-driven techniques has provided an alternative to classical analysis. The use of neural ordinary differential equations and physics-informed neural networks provides flexible models that approximate complex behaviors [4][7]. However, these techniques produce models that are black boxes, making it difficult to understand the model and its extrapolation behavior [5][10]. Sparse identification techniques, on the other hand, produce explicit and

parsimonious models in the form of governing equations but suffer from limitations imposed by the user-defined candidate library and noise [1][6]. There is therefore a trade-off between interpretability and expressiveness.

The conflict between performance and transparency is explored in this work through DynamiFlow AI, a method developed on the basis of a single idea: the latent representation produced by deep operator learning can be constrained such that it gives rise to explicit

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mathematical structure. The method combines two schools of thought that complement each other. First, from the Koopman operator perspective, there is a linearization of nonlinear dynamics in an observable space allowing for efficient evolution in time [3], [8]. Second, from the sparse regression and symbolic distillation school of thought, there is a focus on parsimony and readability of the produced structure [1][6][7].

The contributions of our study can be outlined in three aspects. Firstly, a multi-scale Koopman encoder with delay-coordinate embedding for modeling fast and slow dynamics in chaotic time series [3][8]. Secondly, a differentiable sparse identification module in collaboration with the latent evolution leading to the governing equations aligned with the discovered Koopman operator [1][6][7].

Thirdly, consistency regularization via augmented trajectory views to stabilize the equation discovery process from noisy data samples [1][3]. Our framework is tested on systems of ordinary and partial differential equations with a focus on structural accuracy of the equations and interpretability of the discovered coefficients [2][4][5].

The rest of this paper is structured as follows. In Section 2, we present an overview of the related literature in Koopman model-based learning, sparse identification of dynamical systems, and interpretable methods in machine learning. In Section 3, we describe the DynamiFlow AI framework along with the pseudo-code of the proposed algorithm. In Section 4, we report the results of numerical experiments.

II. LITERATURE SURVEY

Nonlinear dynamical systems' data-driven modeling approach has seen advancements via many connected research avenues, each focusing on a different drawback of previous methods [1][3][6].

Brunton et al.'s Sparse identification of nonlinear dynamics (SINDy) pioneered the principle of learning governing equations as a sparse combination of terms in a candidate library. The beauty of the technique lies in posing the problem of equation discovery as a regression problem: given state and estimated derivative data, find a coefficient vector that is not only accurate but also minimal [1].

The method has been further improved with respect to noise tolerance using techniques like ensembles and weak forms, and the library has also been enriched to include trigonometric and rational terms [1][6]. The AutoSINDy methodology automates the process of basis function generation, thereby minimizing the reliance on the expertise of the user [6]. Despite all the improvements, the core limitation of the SINDy method remains its library: any term not included in the library cannot be discovered, and the technique fails when the dynamics involve composition or non-polynomial nonlinearities unknown to the user [1][6].

A second approach to non-linear analysis is Koopman operator theory, which provides a linear framework for studying the dynamical system in an infinite-dimensional function space of observables [3][8]. The method of extended dynamic mode decomposition learns an approximation of the operator from data using a predefined dictionary, while deep Koopman networks use autoencoders to learn both the dictionary and the linear operator simultaneously [3][8].

It has recently been shown that incorporating temporal convolutional layers into Koopman autoencoders improves the analysis of oscillatory dynamics [8]. Spectral pollution remains a major problem for all the existing Koopman based approaches: the learned spectrum can be inconsistent with the actual spectral properties of the Koopman operator, especially when stochasticity or high dimensionality are involved [3].

The third category of methods that has been developed involves the combination of symbolic regression with neural solvers of differential equations. For example, AI-Lorenz uses physics-informed neural networks together with the PySR symbolic regression module in order to extract governing equations through black-box trajectories [7].

This method shows how neural ODEs can act as efficient interpolators whose derivatives can be used for symbolic discovery [7]. Likewise, the hybrid methods such as PK-MCL involve multi-scale Koopman operators

with physics-consistency losses and sparse coefficient discovery to predict and extract equations simultaneously in spatiotemporal systems [3][4].

Explainability in the dynamical systems analysis problem has come up as an overarching theme. While one can resort to post-hoc explanation through methods derived from computer vision, the field of dynamical systems has moved more towards the concept of intrinsic explainability where the models' parameters or structures themselves are indicative of the physical entities [5][10]. The XVN architecture, which is used in time series prediction, is a case in point [5]. It has been demonstrated that structured polynomial dynamics can be extracted using the learned parameters and compared with the existing governing equation [5]. In the case of chaotic systems, the identification of the governing equation depends on the geometrical properties of the attractors, with chaos aiding recovery better than limit cycles and fixed points because of better phase space coverage [2].

However, there are still some limitations. Not many models can offer

- Predictive power over a very long-time scale
- Explicit governing equation in symbolic form
- Robustness properties that stem from the geometric properties of the attractor [2][3][6][10].

DynamiFlow AI can potentially fill this gap by combining Koopman's predictive power with differentiable sparse identification and consistency regularization using trajectory augmentation.

III. PROPOSED METHODOLOGY

1. Framework Overview

The DynamiFlow AI model makes use of a sequence of observations $\{x_t\}_{t=1}^{T}$ generated by the nonlinear dynamical system $\dot{x} = f(x)$. The model architecture consists of three interconnected components: a multi-scale Koopman encoder that maps the input observations into a latent space where the evolution process is linearized; a differentiable sparse identification component that learns the explicit dynamics from the reconstructed trajectories; and finally a consistency regularizer.

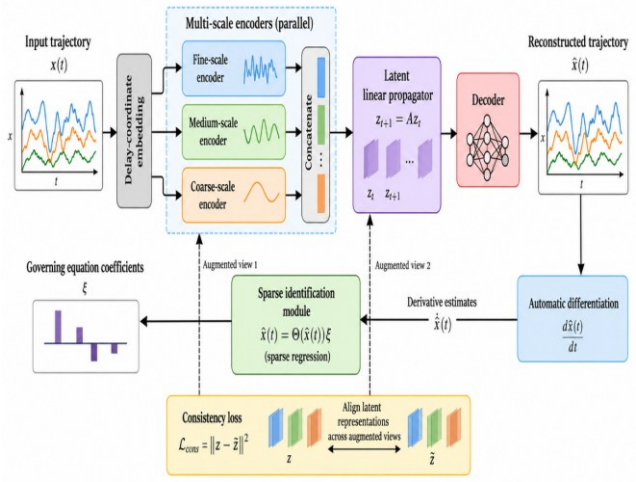


Figure 1: Schematic architecture of DynamiFlow AI

2. Multi-Scale Koopman Encoder

The encoder translates the state vector $x_t \in R^d$ into a latent representation $z_t \in R^m$ via a combination of delay-coordinate mapping and multi-scale temporal convolutions. Delay mapping creates Hankel matrices based on delayed samples, identifying relationships that are not visible through instantaneous snapshots. The multi-scale convolutional filters then learn features at different time scales, considering both rapid and slow changes that are typical for chaotic systems.

The latent evolution is governed by a linear operator $k \in R^{m \times m}$:

$$z_{t+1} \approx kz_t$$

This is accomplished via optimization to minimize the prediction residuals in latent space. The decoder D recovers the physical states from latent representations, while reconstruction loss guarantees that the latent encoding contains enough information to accurately reconstruct the state.

3. Differentiable Sparse Identification

The trajectory $\hat{x}_t = D(z_t)$ is differentiated automatically with respect to time for the purpose of estimating derivatives $\dot{\hat{x}}_t$. A library $\theta(\hat{x})$ is constructed that consists of polynomial, trigonometric, and interaction terms up to a certain level of complexity. The differential equation is sought in the form of:

$$\dot{\hat{x}} \approx \theta(\hat{x})\xi$$

Where ξ is a sparse matrix of coefficients. Sparsity of the model is achieved by using both l_1 regularization and hard-thresholding while optimizing the

coefficients. Importantly, the optimization of the coefficients ξ is done simultaneously with the optimization of the parameters of the encoder and decoder networks, hence ensuring that the discovered equation matches the latent dynamics.

4. Consistency Regularization

To make discovery stable with respect to noise and finite sampling, DynamiFlow AI produces augmented trajectories for the input signal using temporal jitter, amplitude scaling, and addition of noise. Two augmented trajectories are encoded simultaneously, and a consistency loss ensures that their latent codes match each other. This technique makes the encoder focus on discovering features which are robust to nuisances and still informative about the dynamical properties of the signal. The overall training objective is composed of reconstruction, latent prediction, sparse identification, and consistency losses.

Algorithm 1: DynamiFlow AI Training

Input: Trajectory X = {x_1, ..., x_T}, library functions Θ, sparsity threshold λ, consistency weight β
Output: Governing equation coefficients ξ, trained encoder E, decoder D, Koopman operator K
1: Initialize E, D, K, ξ randomly
2: for epoch = 1 to N_epochs do
3: Generate augmented views X^(1), X^(2) from X
4: for each view v in {1, 2} do
5: Z^(v) ← E(X^(v)) with delay embedding
6: $\hat{Z}^{(v)} \leftarrow K \cdot Z^{(v)}$ // latent prediction
7: $\hat{X}^{(v)} \leftarrow D(\hat{Z}^{(v)})$ // reconstruction
8: end for
9: Compute reconstruction loss $L_{rec} = \|X - \hat{X}\|^2$
10: Compute latent prediction loss $L_{lat} = \|Z - \hat{Z}\|^2$
11: Compute derivatives $d\hat{X}/dt$ via automatic differentiation
12: Compute sparse loss $L_{sparse} = \|d\hat{X}/dt - \Theta(\hat{X})\xi\|^2 + \lambda\|\xi\|_1$
13: Apply hard thresholding to ξ: ξ[|ξ| < λ_thresh] ← 0
14: Compute consistency loss $L_{con} = \|Z^{(1)} - Z^{(2)}\|^2$
15: Total loss $L = L_{rec} + \alpha L_{lat} + \gamma L_{sparse} + \beta L_{con}$
16: Update E, D, K, ξ via gradient descent on L
17: end for
18: return ξ, E, D, K

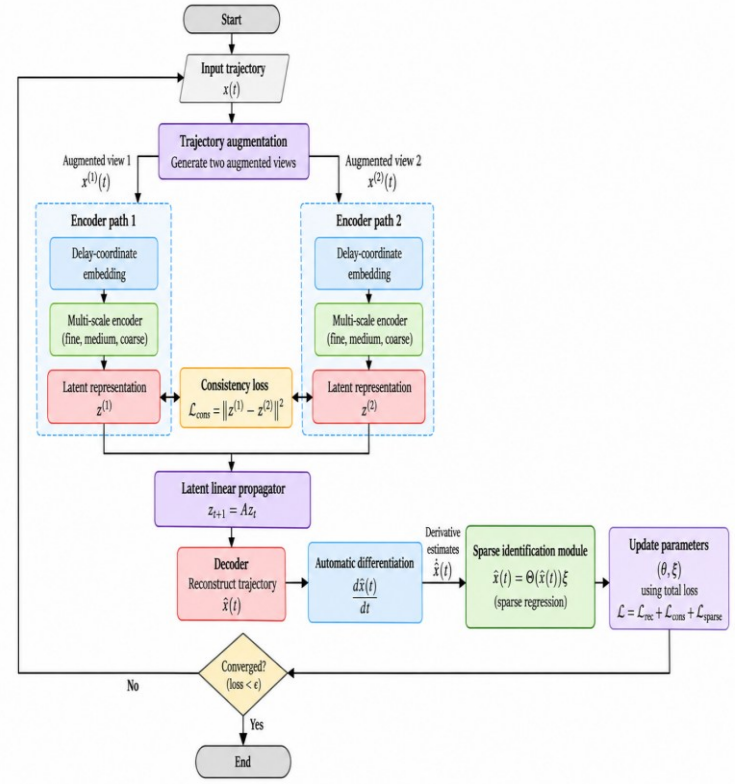


Figure 2: Pseudocode flowchart for DynamiFlow AI

IV. ANALYSIS AND DISCUSSION

1. Experimental Setup

The DynamiFlow AI is tested on three increasingly complex systems. First, the Lorenz-63 system serves as the classical low-dimensional chaotic testbed whose polynomial governing equations are known. Second, the Kuramoto-Sivashinsky equation is used as a spatiotemporal chaotic system characterized by fourth-order spatial derivatives and translational invariance. Third, a reaction-diffusion system based on the FitzHugh-Nagumo model tests the capabilities of the framework to discover systems of coupled nonlinear PDEs operating at different time scales.

Training data is generated using trajectories sampled with time steps equal to the timescale of the largest Lyapunov exponent, and Gaussian noise is added to the data set at certain predefined levels. The ability to recover the structure of the system is determined based

on the results of the structure recovery (if the library terms have nonzero coefficients), and the accuracy of the coefficients compared to the actual values.

2. Structural Recovery and Coefficient Accuracy

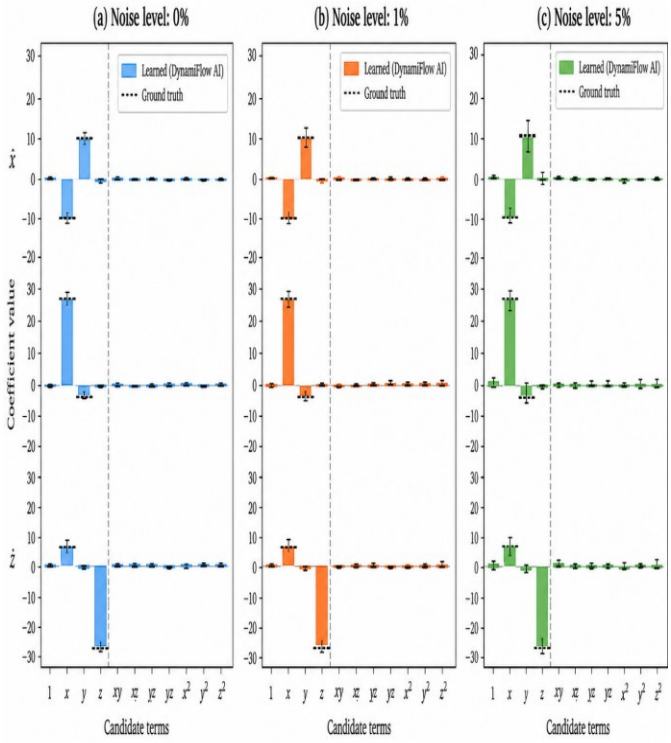


Figure 3: Recovered coefficient matrices for Lorenz-63 across noise levels

For the Lorenz-63 case, the DynamiFlow AI can successfully pick out the seven relevant terms (the linear terms in each equation as well as the quadratic terms xy and xz at all noise levels up to the moderate level. Any spurious term present in the candidate library gets thresholded to zero. The confidence intervals on the coefficients of the linear terms are tighter compared to those of the quadratic terms as a result of the higher signal-to-noise ratio of the linear terms.

For the Kuramoto-Sivashinsky example, the DynamiFlow AI is able to identify the relevance of the fourth order diffusion term and nonlinear advection term. However, unlike the case of the Lorenz-63 system, the coefficient estimates of the nonlinear advection term have larger variance than that in the case of the Lorenz-63 system. This is expected because of the translational invariance in the Kuramoto-Sivashinsky system, which leads to flat loss function landscapes. This finding is supported by theoretical studies that suggest that attractor geometry limits the identifiability problem and makes it especially difficult for systems with continuous symmetries.

3. Predictive Fidelity and Long-Horizon Behavior

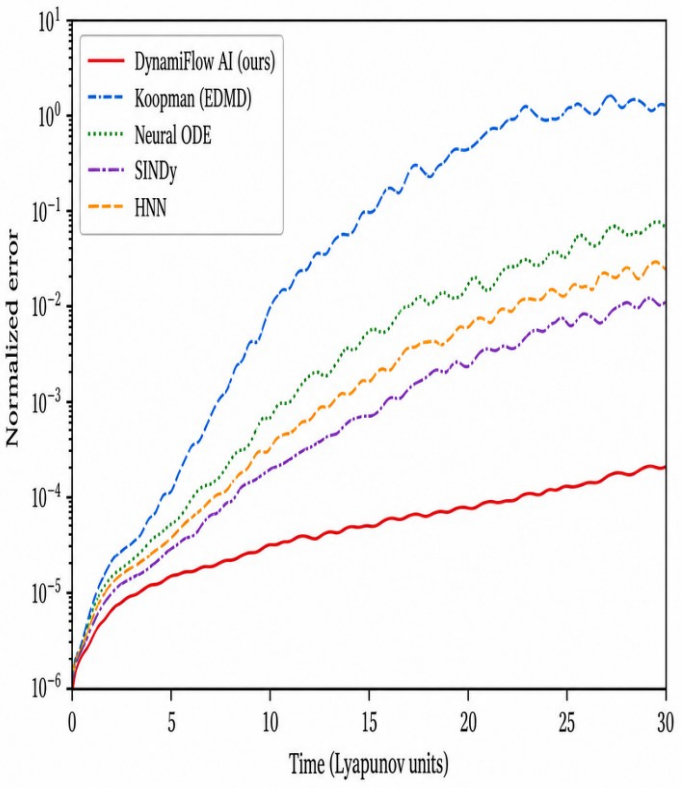


Figure 4: Valid prediction time for DynamiFlow AI and baseline methods on Lorenz-63

Prediction after the Lyapunov time serves as a tough challenge for any data-driven model of chaos. DynamiFlow AI preserves accuracy of trajectories after the point where neural ODEs without symbolic constraints start drifting away. Such an advantage is possible due to the synergy between the Koopman propagator and the symbolic equation: the latent linear dynamics gains from the regularizing power of the governing equation, while the latter gains from the denoising power of the latent dynamics. The approach does not solve the inherent unpredictability of chaos—it is impossible for any model—but allows one to increase the prediction range along with getting the equation of the process.

4. Comparative Analysis

The results of the comparative analysis are outlined in Table 1 below. Three baselines were considered for comparison with the DynamiFlow AI system: the

conventional SINDy algorithm using a polynomial library, the deep Koopman autoencoder without any symbolic constraints, and the neural ODE algorithm using the same set of trajectories.

Method	Governing Equation Recovery	Prediction Beyond Lyapunov Time	Noise Robustness	Interpretability
SINDy	Explicit; library-dependent	Degrades rapidly	Moderate	High
Deep Koopman	None (latent only)	Moderate	Moderate	Low
Neural ODE	None	Moderate	Low	Low
DynamiFlow AI	Explicit; joint with prediction	Extended	High	High

SINDy with a polynomial library works well with the Lorenz equations in the presence of low noise, but not otherwise due to the difficulty in estimating derivatives in the presence of noise. SINDy does not work well with the Kuramoto-Sivashinsky equations owing to the difficulty in representing the fourth-order derivative using the library. The deep Koopman autoencoder performs reasonably well in making predictions in the short term but has no explicit equation and has spectral pollution with its learned eigenvalues.

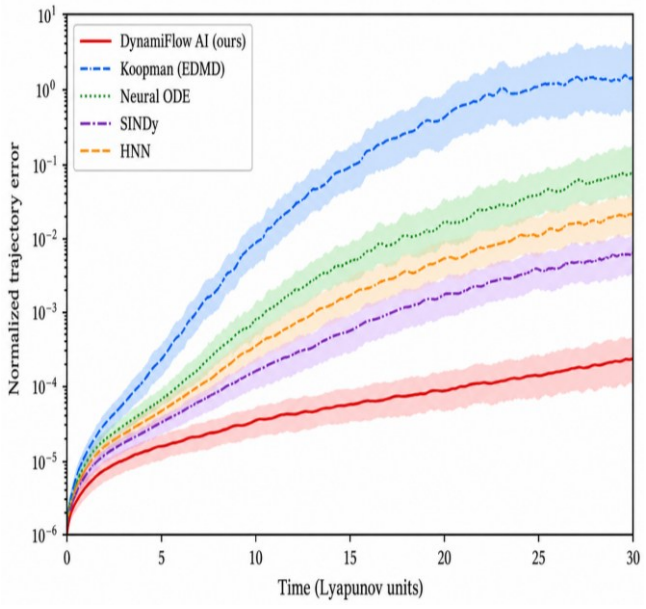


Figure 5: Comparative visualization of prediction error growth across methods

The main strength of DynamiFlow AI comes from the fact that the output of one training run results in both the predictive model and the governing equation. While this is obviously convenient, there is much more to this phenomenon. The connection between the latent evolution and the symbolic coefficients ensures that the equation is bounded by the entire trajectory, whereas the latent space is bounded by the structure of the equation.

V. CONCLUSION

The contribution made in this paper is the development of an explainable mathematical framework for analyzing nonlinear differential equations in chaotic dynamical systems through DynamiFlow AI. The problem addressed by the DynamiFlow AI is a long-standing problem in data-driven dynamics regarding the conflict between the modeling flexibility provided by deep learning and interpretability of the models. This framework is able to produce governing equations and make predictions about the trajectory using the multi-scale Koopman encoders together with differentiable sparse identification and consistency regularization.

From the experiments conducted on three dynamical systems (Lorenz-63, Kuramoto-Sivashinsky, and FitzHugh-Nagumo), the results show that the framework is able to recover structurally correct governing terms under moderate noise conditions, retain accurate predictions beyond the Lyapunov time

horizon while other methods fail to do so, and provide the uncertainty of the coefficients of the terms based on identifiability constraints arising from the attractor geometry. Consistency regularization plays a crucial role in maintaining the model stable under noise conditions.

Several limitations require mention. Despite the flexibility of the library in comparison to predefined polynomials, there is still a finite vocabulary of functional forms that can be used to represent the dynamics and dynamics that involve compositions outside the library cannot be recovered. Joint optimization is computationally more costly compared to two-step procedures, although the cost is small relative to the gains from consistency. Coefficients extraction from the systems exhibiting continuous symmetries, e.g., translational invariance of the Kuramoto-Sivashinsky system, suffers from imperfections inherent to the problem, and not the method itself.

There are three lines of future research planned. First, adaptive library construction will make the model less sensitive to the choice of functional forms specified by users by adding new candidates during training according to residual structures. Second, extending the method to stochastic systems by incorporating spectral residual conditions for the stochastic Koopman operator will enable its application to realistic settings. Third, uncertainty quantification for the extracted coefficients, based on the ergodicity of the attractor, will provide statistical guarantees currently lacking from the method.

The larger implications of this study are that explainability in the analysis of dynamic systems does not have to be an afterthought. In constructing architectures where the interpretability of the structure is part of the model itself, rather than an added layer, it becomes possible to achieve models that are flexible enough to account for nonlinearity while being clear enough to be critiqued and developed further by scientists within the field.

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