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Mathematical Modeling and Optimization in Sustainable Electrical Engineering: Recent Trends and Future Directions

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Abstract- The movement towards sustainability in electrical engineering requires precise mathematical models and optimization methods for handling the variability of the renewable resources, scheduling storage systems, and assuring economic feasibility. This work provides an organized review of recent developments in optimization research related to hybrid system sizing, unit commitment with renewable generation, demand-side management, and EVs charging schedule optimization. A combined approach that uses mixed-integer nonlinear programming for the unit commitment and particle swarm optimization for optimal power flow problem solving serves as an example of one such framework. Analysis of recent literature shows that hybrid optimization methods surpass simple methods in constraint handling and multi-objective optimization quality, and reinforcement learning has proven useful for scheduling and curtailing reduction. The table below compares the performance of different algorithms in recent research. Finally, the future direction of development in this field includes hybridization, data-based, and multi-scale optimization.

Keywords- Sustainable Electrical Engineering, Mathematical Optimization, Renewable Energy Integration, Hybrid Metaheuristics, Unit Commitment, Demand-Side Management

I. INTRODUCTION

The field of sustainable electrical engineering has moved from being an area of focus for only a few to becoming a major design constraint in generation, transmission, distribution, and consumption sectors [1][6]. The increasing incorporation of intermittent forms of renewable energy, primarily photovoltaic and wind energy, into the power grid system has created a degree of variability in operation, which the existing deterministic optimization tools were not meant to handle [1][6]. Additionally, the rise in electrification of transportation systems along with the deployment of distributed energy sources has increased the scope of decision making in terms of optimization [4][10].

The optimization scenario in sustainable electrical engineering faces inherent conflicts. First, there is an opposition between economy and ecology since the lowest levelized cost of energy may imply systems with low use of storage devices or curtailment of renewables, while reduction of the carbon footprint involves oversizing of renewable generation at the cost of lower economy [1][8]. Second, reliability constraints create conflicts between reliability and cost minimization, especially when optimizing battery sizes in the case of isolated microgrids since these values determine costs and amounts of unmet

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load [8][9]. Third, decision-making processes differ greatly by time scales from second-scale frequency control to seasonal storage control [6][7].

There have been three main approaches adopted by recent methodological improvements to address these problems. The first approach is that of hybridization, in which multiple algorithms are used together so as to benefit from the strength of each one of them [2][3]. A particular case in point is a hybrid use of mixed-integer non-linear programming for the day ahead unit commitment along with the particle swarm optimization for improving the power flow results, resulting in near-total violation minimization at minimal cost [2][3]. The second approach is that of inclusion of a learning component. The reinforcement learning algorithm has been used for real time scheduling of the optimal power flow and unit commitment problems in a single Markov decision process, leading to a large reduction in the curtailment of renewables with constraint satisfaction equivalent to physics-based solvers [7].

Despite all this advancement, there are still a number of deficiencies that exist. First, a lot of researches that have been published on optimization concentrate more on operational scheduling or capacity planning decisions without any combination between the two [6]. Secondly, comparative research among different algorithms has not been conducted extensively since most of the papers benchmark with very few alternatives through domain-specific test cases [1][6]. Thirdly, social and equity considerations in optimization objective functions are very new especially for rural areas where multi-energy systems would benefit [1].

The gaps noted above have been addressed in this paper through consolidation of recent advances in methodology into an integrated approach along with a systematic comparative assessment. The contribution is in three aspects. First, there is a compilation of literature in four optimization areas—hybrid system sizing, scheduling, demand side management, and electric vehicles [1][4][6]. Secondly, a complete methodological model has been provided for hybrid unit commitment-optimal power flow approach along with system and algorithmic diagram models [2][3]. Thirdly, the performance comparison of various algorithms through recent work and future research directions have been synthesized quantitatively. It should be noted that the focus is only on mathematical optimization methods that have sustainability goals.

II. LITERATURE SURVEY

Optimization research relating to sustainable electrical engineering from 2021 to 2026 has grown significantly, and several clear lines of research may be identified in the existing literature [1][6].

There is a large volume of research on the optimal design of hybrid renewable systems comprised of photovoltaics, wind energy, batteries and dispatchable sources [1][6]. Current surveys have indicated the preponderance of metaheuristic algorithms—in particular, particle swarm optimization and differential evolution—for the optimization of the levelized cost of energy in terms of the constraint of loss of power supply probability [1][8]. The multiobjective approach has been formulated for the optimization of grid-connected photovoltaic systems equipped with hybrid storage technologies, resulting in a Pareto frontier between the system cost and curtailment [1][8]. In the rural setting, an interesting case study focused on a multi-energy complementary system that includes biomass, solar and wind energy, where a planning model was defined, having as its objectives life cycle cost minimization, CO2 emissions minimization and primary energy savings; however, rather than fixing the efficiency of the components, the partial load ratio of the equipment was taken into account [1][6]. The incorporation of renewable generators into the process of unit commitment and economic dispatch has spurred many novel methodologies to emerge [2][3]. A mixed integer nonlinear programming approach for unit commitment and PSO-based optimal power flow revealed that schedules initially generated on cost-optimal principles may breach system constraints, while the second phase results in a decrease in violations by over 99% at a negligible cost [2][3]. In addition, microgrid operational scheduling was investigated through the lens of a

chaotic self-adaptive sine cosine algorithm in the context of cost-emission conflict, with Pareto optimal results reported in three scenarios, one of which involved exclusively non-zero emission sources. Even reinforcement learning has entered this field: a planning-enabled approach for reinforcement learning that viewed optimal power flow and unit commitment as a single MDP problem with tree search ahead managed to reduce renewable curtailment by 79% while maintaining similar constraint satisfaction levels as conventional solvers [7].

From the initial static time-of-use based optimization for demand-side flexibility, optimization approaches have evolved into dynamic models which are also redundant awareness based [4]. A multi-objective charging scheduler employing an enhanced multi-objective grey wolf optimizer developed a closed-loop "redundancy-price-behavior" approach, creating dynamic charging prices considering pre-calculated power redundancy for shifting the load towards the renewable rich period [4]. Modified osprey optimization algorithm applied to smart grid-based demand-side management problem with solar photovoltaic integration showed superior performance in reducing peak loads and balancing the load profile [5]. Battery energy storage sizing problem of microgrids has been studied by means of ε-MOPSO, optimizing for both levelized cost and supply reliability probability simultaneously [9].

From these research categories, three clear trends can be observed. Firstly, the use of hybrid and multi-objective modeling strategies has supplanted the former approaches based on single-objective and single-method models [1][6]. Secondly, the connections between operational and strategic decision-making have received increased consideration, but there are few integrated models that capture both aspects simultaneously [2][3][6]. Thirdly, learning-based algorithms are becoming prominent as supplements to classical optimization models, especially within time-limited decision layers characterized by short forecasting periods [7]. There is however no overall comparison of these models in terms of their comparative performance. This will be done in the subsequent section of this paper.

III. PROPOSED METHODOLOGY

Framework Overview

The proposed methodology combines features of the prevalent hybrid approach found in literature, namely a two-level optimization process integrating unit commitment scheduling and optimal power flow refinement. This process is chosen since it specifically targets the issue of constraint violations caused by a lack of consideration of physical laws of the network in scheduling algorithms – a problem pointed out in contemporary research comparing various methods. The methodology is described on an abstract enough level to be applied to hybrid renewable energy systems.

Figure 1 represents the system architecture in a general way. The architecture includes three layers which are:

- A layer for forecasting and scenario generation that supplies the profiles of photovoltaic generation, wind generation, and load demands one day ahead
- A scheduling layer that is used to solve the unit commitment problem to get the generator commitment and dispatch setpoints
- A refinement layer where the schedule is checked and adjusted based on AC optimal power flow constraints by a meta-heuristic assisted optimization algorithm

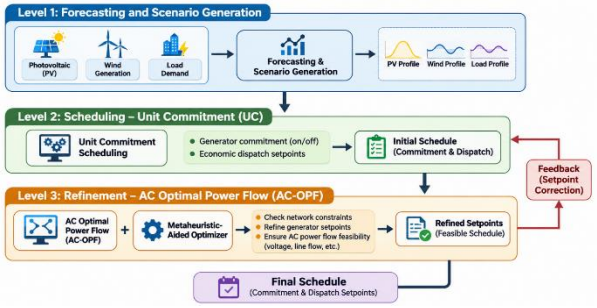


Figure 1. System architecture of the two-stage hybrid optimization framework

Stage 1: Unit Commitment Formulation

The unit commitment problem is mathematically modeled as a mixed integer nonlinear programming

problem. The objective function is to minimize the total cost of operation during a day, which includes the cost of fuels, cost of startups and shut-downs, and costs related to renewable curtailment.

$$min \sum_{t=1}^{T} \left[\sum_{i=1}^{Ng} \left(C_i(P_{i,t}) + SU_i \cdot u_{i,t}^{start} + SD_i \cdot u_{i,t}^{stop} \right) + \lambda \cdot P_t^{curt} \right]$$

where $c_i(\cdot)$ denotes the quadratic cost function representing the cost of fuel consumed by thermal unit $i, u_{i,t}^{start}$ and $u_{i,t}^{stop}$ are start-up and shut-down decision variables respectively, while P_t^{curt} is the penalized curtailed renewable energy at a penalty factor of λ.

Nonlinearities arise from the quadratic cost functions and from the power balance equation whenever storage efficiency is included state-dependently. This is solved as an MINLP problem using branch and bound approach with nonlinear relaxations, and yields a solution that is cost-optimal but violates network-level constraints.

Stage 2: PSO-Assisted Optimal Power Flow Refinement

The schedule obtained from stage one is then fed into an optimal power flow refinement stage. The objective function of the AC-OPF problem is defined as follows:

$$min \left(\sum_{i=1}^{Ng} | P_{i,t}^{adj} - P_{i,t}^{UC} + \rho \sum_{k=1}^{Nc} V_k^{viol} \right)$$

subject to the complete nonlinear power flow equations, voltage magnitude constraints, and line flow constraints. The control variables for this problem will be perturbations from the set points in the schedule derived at Stage 1. Particle swarm optimization will be applied to investigate the solution space, where a particle is defined as a set of set point perturbation values. Newton-Raphson power flow is used to evaluate the candidate solutions, with the exact constraint violations calculated for every candidate.

Figure 2 depicts the flow chart of the process of refinement. In this algorithm, the process begins with the initiation of the particles near the Stage 1 schedule, then proceeds with the iterations of velocity and position updates of particles, candidate evaluations by Newton-Raphson power flow, and ends when the violation penalty exceeds some certain threshold or the number of iterations becomes maximum.

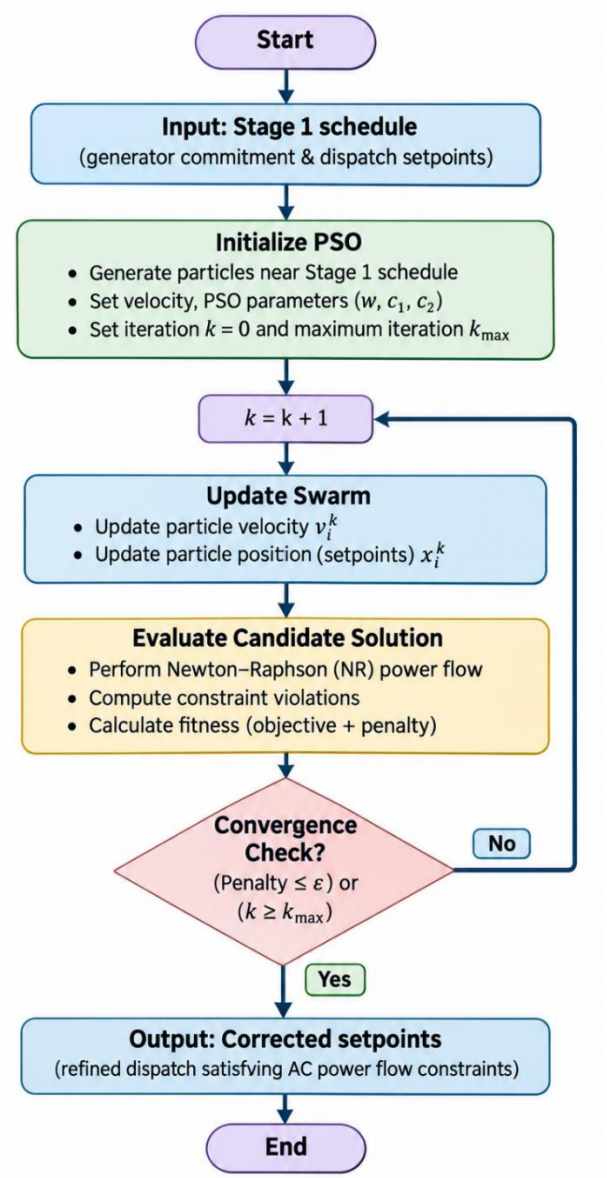


Figure 2. Algorithmic flow of the PSO–Newton–Raphson refinement stage

Demand-Side and Storage Coordination Module

The two-step framework is generalized to include the concept of demand-side flexibility and coordinated storage control by introducing it as an additional post-processing step. The flexibility of load shifting is formulated through a price-sensitive demand curve, whereas the control of battery storage is done in such a way that the variance of residual net load is minimized after unit commitment and power flow optimization is done. The post-processing step employs a rule-based heuristic approach that was motivated by the concept of redundancy-aware pricing. In brief, charging is done whenever the optimized schedule has a higher production level of renewable energy, while discharging occurs when there is peak net load.

Implementation Considerations

The structure is intended to be implemented using Python programming language with the Pyomo optimization solver for the MINLP part and the proprietary PSO algorithm for the refinement part, interacting with MATPOWER for solving the power flows. The decoupling of the stages allows for the usage of commercially available solvers for the MINLP problem and a meta-heuristic for solving the non-convex optimization problem. The computational effort lies within the PSO-NR cycle when each particle needs a power flow solution at each iteration; hence, parallelization is possible for larger systems.

IV. ANALYSIS AND DISCUSSION

Constraint Satisfaction and Cost Trade-off

One of the most important results common to all recent research into hybrid optimization is that separating the scheduling process from the physical constraints of the network creates schedules that are economically viable yet physically unviable. In the hybrid optimization model MINLP–PSO–NR implemented on the modified IEEE 30-bus network with thermal generators, PV stations, and pumped hydro storage, the initial MINLP schedule had an operating cost of 8,876.92 cost units but incurred a constraint violation penalty of 302,210.9. The subsequent refinement resulted in over 99% reduction in constraint violations, with the new penalty being just 2,128.49, while the operating cost increased slightly to 9,280.94. This is evidence of the "price of feasibility" in hybrid renewable energy scheduling, which amounts to about 4.5% increase in costs in order to satisfy the constraints. Figure 3 illustrates the above point visually.

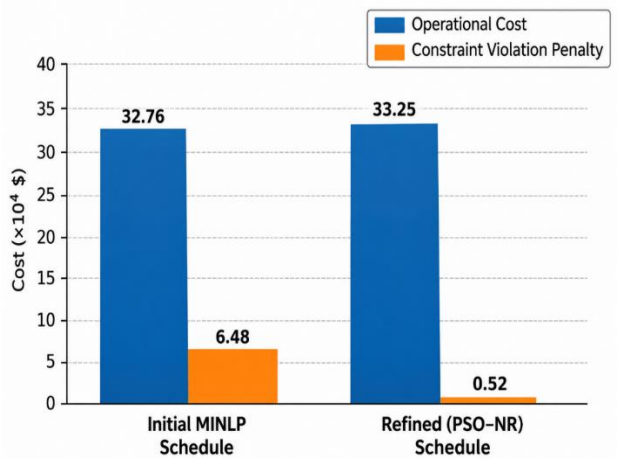


Figure 3. Cost–penalty trade-off before and after PSO–NR refinement

Multi-Objective Algorithmic Performance

Evidence from various studies indicates that algorithm choice significantly impacts the quality of Pareto frontiers in the case of multi-objective formulations. In a microgrid scheduling problem where there were cost and emissions minimizations as the objectives, the chaotic self-adaptive sine cosine algorithm attained cost and emission values of 590.45 €ct and 337.28 kg respectively in a full-renewable case scenario; and this was superior to the standard sine cosine algorithm in all the three test cases considered. This superiority was as a result of chaotic self-adaptation mechanisms which increase exploration without compromising on convergence. In battery sizing for microgrids, Pareto fronts obtained through ε-MOPSO revealed that minimizing levelized cost and maximizing reliability at the same time was possible, with the best combination reducing dependency on the grid while ensuring that the probability of supply remains above certain thresholds. Figure 4 shows a typical example of a Pareto frontier for the cost-emissions trade-off.

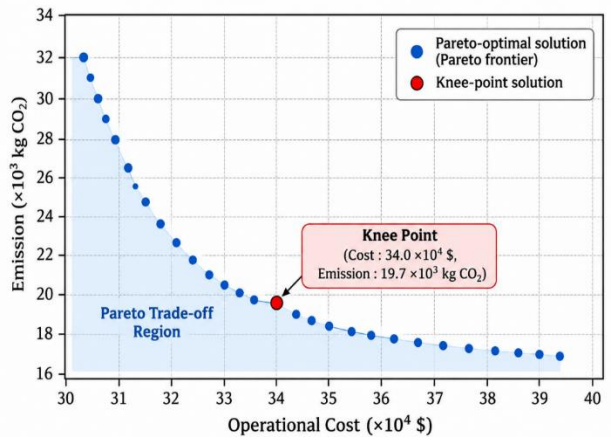


Figure 4. Illustrative Pareto frontier for cost–emission trade-off in microgrid scheduling

Curtailment and Renewable Utilization

Reinforcement learning methods have shown especially good results for curtailment reduction problems. The planning-based RL algorithm achieved an 79% curtailment reduction from the baseline scheduling approach, while satisfying constraints comparable to those satisfied by physics-based solvers. This result is impressive since curtailment reduction usually has secondary importance in the classical formulation. By employing the short-term forecasting using the tree search procedure, the RL algorithm is better equipped to foresee curtailment events compared to the myopic dispatching rule. However, according to the same paper, there were concerns regarding sensitivity to forecast quality and topology changes. In the case of demand side management, the redundant-based EV charging method showed that dynamic pricing when linked to precalculated power redundancy could lead to cost reduction for users while increasing renewable use. This technique works by determining periods where there is excess production of power through renewables compared to the local consumption of power, and then determining the price charged to encourage consumption of the surplus during such periods. Figure 5 gives the conceptual relation between the power redundancy, dynamic price signal, and charging load shift.

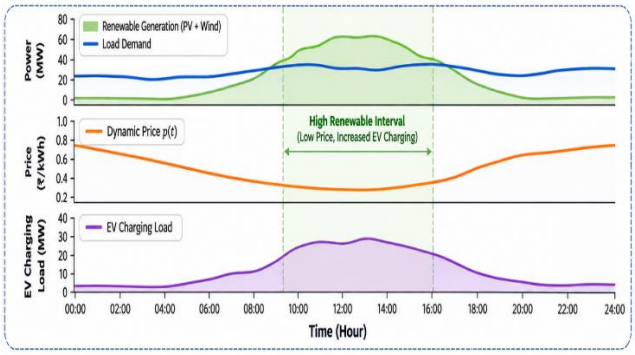


Figure 5. Conceptual illustration of redundancy-price-behavior coupling for EV charging

Comparative Analysis

Table I presents the quantitative results from selected recent studies within the categories of optimization discussed. The presentation of this table is organized according to the goal of the optimization, the methodology employed, and the outcome variables used. One must bear in mind that direct comparison of numbers between studies is limited by the differences in test cases, cost years, and assumptions.

Table 1. Comparative Analysis of Recent Optimization Studies

Study Focus	Optimization Objective(s)	Algorithm(s)	Key Quantitative Outcome
PV-Hydro-Battery sizing	Minimize LCOE and LPSP	PSO, Differential Evolution	LCOE = 0.062 USD2024/kWh; LPSP = 0.05; DE outperformed PSO
UC-OPF with PV and PHS	Minimize cost and constraint violation	MINLP + PSO-NR hybrid	Penalty reduced from 302,210.9 to 2,128.49; cost +4.5%

Study Focus	Optimization Objective(s)	Algorithm(s)	Key Quantitative Outcome
Rural multi-energy system	Minimize cost, emissions; maximize energy savings	Multi-objective planning model	Part-load modeling improved realism vs. constant-efficiency assumption
EV charging scheduling	Minimize cost, maximize renewable utilization	Improved multi-objective grey wolf optimizer	Redundancy-price coupling reduced charging cost and increased renewable absorption
Microgrid battery sizing	Minimize LCOE, maximize reliability	PSO and ε-MOPSO	Pareto-optimal configurations reduced grid dependence while meeting reliability targets
Microgrid scheduling	Minimize cost and emissions	Chaotic self-adaptive sine cosine	Cost 590.45 €ct, emissions 337.28 kg in renewable-heavy scenario
Real-time scheduling	Minimize curtailment, maintain constraints	Planning-based reinforcement learning	79% curtailment reduction; constraint satisfaction comparable to physics-based solvers

There is an observable trend from the above table, where the dominant approach is that of hybrids and multiple objective formulation in recent studies showing higher performance. The greatest advances in terms of quantitative improvements are observed from the combination of different decision levels which were traditionally considered independently. They include scheduling and power flow, pricing and redundancy forecasting, learning and lookahead planning.

Limitations and Open Challenges

However, three constraints qualify any optimism about recent outcomes. First, the increased complexity involved in hybrid algorithms and multi-objective techniques increases exponentially with the scale of the system, meaning that the PSO-NR optimization process, which works well for the IEEE 30-bus network, requires significant parallelism for practical transmission systems. Second, uncertainties are considered mostly using the scenario-based approach without any use of distributionally robust or stochastic optimizations. Third, the socio-economic aspects of sustainability, such as equity of access, affordability for poor consumers, and social acceptance, remain largely ignored in terms of the objectives function.

V. CONCLUSION

The current research has critically discussed and summarized the trends in mathematical modeling and optimization for sustainable electrical engineering, particularly regarding hybrid renewable energy systems, scheduling optimization, coordination between consumers, and EVs. Several findings can be stated.

In particular, the literature reveals that there is a trend towards hybrid optimization models for

addressing issues related to variability in renewables and network limitations. The two-stage MINLP-PSO-NR model represents one of the examples of such trend, since the cost of feasibility – 4.5% in this case – is lower than the costs of violating any of the constraints.

The second trend is that multi-objective models have evolved to the level where Pareto fronts of practical significance can now be identified for trade-offs of cost-emission and cost-reliability. Advances in algorithm design, especially in terms of chaos-based self-adaptive methods and ε-dominance, have helped improve both the spread and the convergence of Pareto sets. But the decision and interpretation stages where Pareto fronts are transformed into feasible action plans are still relatively underdeveloped; most works do not include such frameworks for choosing non-dominated solutions.

Thirdly, learning techniques, especially those based on reinforcement learning with the planning capability, have unique benefits when applied to real-time and quasi real-time decision problems. The 79% curtailment reduction achieved with planning-based RL is a sign that such techniques can supplement optimization through accounting for the short-term stochastic component inadequately captured by scenario approaches. The combination rather than replacement of learning and optimization seems to be the most promising path.
Four areas of research need particular attention in future work. Multi-timescale integration needs modeling techniques that can co-optimize across planning, dispatch, and control timescales without introducing decomposition error. Uncertainty-aware optimization needs to transition from scenario-based sampling approaches to distributionally robust techniques that have a guarantee of performance despite the uncertainty in the distribution of disturbances. Data-efficient learning for optimization needs solutions to the sample complexity problem of reinforcement learning (and other learning techniques) in the context of safety-critical systems such as power networks. Finally, equity and distributional criteria need to be properly formalized: sustainability is not merely a technical/economic criterion, and an optimization framework that neglects affordability and accessibility is doomed.

The mathematics of sustainable electrical engineering is steadily growing more advanced, but in the end, its success relies on relevance to the political and social contexts in which power networks operate. The research with the biggest impact will probably be that which does not see optimization as a purely technical task, but as a decision support tool within larger decision-making systems.

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