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Advanced Numerical Methods in Fluid Dynamics Research

Dr Thadakamalla Srinivasulu, Assistant Professor
GDCW (A) Nalgonda

Abstract- The numerical simulation of fluid flows continues to advance through the integration of machine learning, high-order discretizations, mesh-free frameworks, and emerging quantum-inspired algorithms. This review synthesizes peer-reviewed research published between 2025 and 2026, focusing on five transformative methodologies: physics-informed neural networks augmented with lattice Boltzmann kinetics, hybrid high-order formulations for turbulent flows, iterative high-order smoothed particle hydrodynamics, p-adaptive mesh-free frameworks, and quantum-assisted computational fluid dynamics (CFD). Key quantitative advances include two orders of magnitude improvement in smoothed particle hydrodynamics accuracy, mean absolute error reduction by a full order of magnitude in neural network solvers, computational cost savings up to 50% through p-adaptivity, and machine learning acceleration of CFD simulations by up to 10,000 times. These developments collectively indicate a paradigm shift toward hybrid, adaptive, and cross-paradigm numerical methods that address the longstanding trade-offs between accuracy, stability, and computational cost in fluid dynamics research.

Keywords: computational fluid dynamics; physics-informed neural networks; high-order methods; mesh-free methods; quantum computing; turbulence modeling.

I.INTRODUCTION

The accurate and efficient simulation of fluid flows remains a central challenge across engineering and the physical sciences. Traditional numerical methods such as finite differences, finite volumes, and finite elements have served as the workhorse of CFD for decades. However, their limitations become pronounced when confronted with complex geometries, high Reynolds numbers, multiphase physics, or inherently multiscale phenomena such as turbulence.

The 2025–2026 period has witnessed a remarkable convergence of classical numerical analysis with artificial intelligence, high-order discretizations, and even quantum computing paradigms. These advances are not merely incremental but represent a fundamental rethinking of how fluid equations are approximated, solved, and accelerated.

This review focuses exclusively on peer-reviewed research published in 2025–2026, drawing from journals and conference proceedings indexed in Scopus. The emphasis is on numerical methods that have demonstrated substantial improvements in accuracy, stability, or computational efficiency—often by orders of magnitude—over their predecessors.

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II. AI-INFUSED NUMERICAL METHODS: FROM PINNS TO LEARNED TURBULENCE MODELS:

Hybrid Physics-Informed Neural Networks:
Physics-informed neural networks (PINNs) embed governing physical laws directly into the loss function of a neural network, offering a mesh-free and data-efficient alternative to conventional discretizations. Despite their promise, standard PINNs suffer from poor stability and accuracy at high Reynolds numbers. A 2025 study introduced PINN-MRT, a hybrid architecture that integrates the multi-relaxation-time lattice Boltzmann method (MRT-LBM) with PINNs. The model employs a dual-network architecture that separately predicts macroscopic conserved variables and non-equilibrium distribution functions.

In inverse problems, PINN-MRT remains stable at Reynolds numbers up to 5000 with parameter inversion errors below 15%, whereas standard PINNs fail at a Reynolds number of 1000 with errors exceeding 80%. In purely physics-driven forward problems, PINN-MRT provides stable solutions at a Reynolds number of 400 while comparable models completely collapse.

Complementing this approach, the T-PINN-LBM framework combines a mesoscopic lattice Boltzmann model with a tanh robust weight initialization derived from fixed-point analysis. This method mitigates activation and gradient decay in deep networks, reducing the mean absolute error by one order of magnitude at the same network depth while maintaining stable convergence in deeper networks.

Machine Learning for Turbulence and Reduced-Order Modeling:
The integration of machine learning (ML) with CFD has accelerated dramatically. A comprehensive 2025 review documented that ML techniques can accelerate simulations by up to 10,000 times in certain cases while maintaining or improving accuracy. Specifically, models employing learned interpolation achieve 40- to 80-fold computational speedups while matching the accuracy of baseline solvers at a resolution 8 to 10 times finer. Fourier Neural Operators demonstrate inference times three orders of magnitude faster than conventional PDE solvers for the Navier–Stokes equations.

In the domain of reduced-order modeling, a parametric reduced order model combining Proper Orthogonal Decomposition (POD) with Radial Basis Function-generated Finite Differences (RBF-FD) meshless simulations was developed in 2025. This approach is particularly suitable for thermofluid dynamic problems on parametric domains, as it handles complex geometries and large deformations without the need for mesh generation or refinement.

III. HIGH-ORDER AND MESH-FREE ADVANCES:

Hybrid High-Order Formulations for Turbulence:
Turbulent flow simulations require both high spatial resolution and robust numerical stability. A 2025 study proposed a Hybrid High-Order (HHO) formulation of the incompressible Navier–Stokes equations specifically designed for turbulent flows. The formulation features pressure-robustness, cell-by-cell mass conservation to machine precision, robustness in the inviscid limit, and implicit high-order accurate time stepping with local time step adaptation.

By enabling static condensation of both velocity and pressure, the method significantly reduces memory footprint while maintaining high-fidelity accuracy for challenging test cases such as the Taylor–Green vortex at Reynolds 1600.

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Iterative High-Order Smoothed Particle Hydrodynamics:

Smoothed particle hydrodynamics (SPH) is a Lagrangian mesh-free method well-suited for large deformation problems and free-surface flows. However, achieving consistent high-order accuracy under irregular particle distributions has remained a fundamental challenge. A 2025 study introduced an iterative high-order SPH framework that systematically improves the accuracy of gradient and Laplacian operators through multiple layers of Taylor expansions.

The proposed method achieves up to fourth-order convergence even with irregular particle arrangements and improves simulation accuracy by two orders of magnitude compared to conventional SPH formulations, while avoiding high-order kernel functions and large matrix systems.

p-Adaptive High-Order Mesh-Free Frameworks:

A p-adaptive high-order mesh-free framework was developed in 2025 for accurate and efficient simulation of fluid flows in complex geometries. The method constructs high-order differential operators locally for arbitrary node distributions using linear combinations of anisotropic basis functions. A dynamic p-refinement strategy adjusts the polynomial order at each node based on local error estimates of the Laplacian operator. For the test cases studied, the method exhibits potential to save up to 50% of computational costs while maintaining the specified level of accuracy.

IV. CROSS-VALIDATION AND QUANTUM PERSPECTIVES:

Mesh-Free Solvers for Porous Media Flows:

A 2025 cross-validation study compared two mesh-free CFD solvers for pore-scale fluid flow through porous media: the lattice Boltzmann method with a two-relaxation-time collision term and a direct Navier–Stokes solver under the artificial compressibility limit. Both solvers employed the same h-refined meshless spatial discretization and radial basis function (RBF) method to approximate fields and differential operators.

The results achieved excellent agreement with literature data in terms of drag coefficient and permeability across different porosities, and the simulations were extended beyond the previously reported porosity range.

Quantum and Quantum-Inspired Approaches:

The scalability challenges of conventional CFD are most acute in high-dimensional, multiscale, and turbulent regimes. A 2025–2026 review surveyed advances at the intersection of quantum computing, quantum algorithms, and tensor network techniques for CFD. While fully quantum CFD remains out of reach in the current NISQ (Noisy Intermediate-Scale Quantum) era, quantum-inspired tensor network methods already show practical benefits, including several orders of magnitude reductions in memory and runtime while preserving accuracy.

Hybrid quantum-classical approaches, particularly those using Variational Quantum Algorithms and Quantum Physics-Informed Neural Networks, offer the most promising near-term strategy for achieving scalable CFD solvers.

V. CONCLUSION:

The numerical methods for fluid dynamics have undergone a profound transformation in the 2025–2026 period. Hybrid PINN-LBM architectures have extended stable simulations to Reynolds numbers ten times higher than previously possible, while reducing error by an order of magnitude through

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optimized weight initialization. High-order SPH frameworks have achieved fourth-order convergence under irregular particle distributions, improving accuracy by two orders of magnitude over classical formulations. p-adaptive mesh-free methods have demonstrated computational cost savings of up to 50% through dynamic polynomial order adjustment. Machine learning has accelerated CFD simulations by factors ranging from 40-fold to 10,000-fold across different regimes, while quantum-inspired tensor network methods offer reductions in memory and runtime by multiple orders of magnitude.

Collectively, these advances signal a paradigm shift: the field is moving beyond the classical dichotomy of mesh-based versus mesh-free, low-order versus high-order, or physics-driven versus data-driven. Instead, the most powerful numerical methods of the coming decade will be hybrid frameworks that integrate the strengths of multiple paradigms—mesoscopic kinetic theory with deep learning, high-order discretizations with adaptive refinement, and quantum-inspired compression with classical solvers. These developments are not only accelerating existing engineering workflows but also opening entirely new avenues for scientific discovery in fluid systems previously considered computationally intractable.

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