



Nested Self-Amplifying Hypergraphs: A New Framework for Endogenous Growth

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Abstract- This paper introduces Nested Self-Amplifying Hypergraphs (NSAHs — a completely new mathematical structure for modeling how economies grow from within. Unlike standard models that assume simple, pairwise interactions, NSAHs capture three real-world features: (1 many activities can combine at once (hyper-edges, (2 the output of a process can feed back to strengthen itself (self-amplification, and (3 small-scale processes nest inside larger ones, passing along their gains. We show that such a system naturally generates sustained growth without external shocks. An implementable algorithm is provided, and a synthetic example demonstrates the idea. No part of this framework exists in prior literature.

Keywords: Nested Self-Amplifying Hypergraphs (NSAHs), Economic Growth Theory, Hypergraph Models, Endogenous Growth, Complex Systems, Self-Amplification, Higher-Order Interactions, Network Economics, Economic Complexity, Recursive Dynamics, Hierarchical Systems, Computational Economics.

I. INTRODUCTION

Standard economic growth models — from Solow to Romer — treat production as a function of capital and labor, with interactions assumed to be linear or pairwise. But consider a modern AI lab: producing a useful model requires simultaneous inputs (data, compute, algorithms. The output then improves the lab's own ability to generate more data and better algorithms. Moreover, small experiments (child processes feed into larger training runs (parent processes. This nested, multi-way, self-reinforcing loop is poorly captured by existing mathematics.

Hypergraphs — where edges can connect any number of nodes — have been used in economics before, but never with self-amplification and nesting combined. NSAHs fill this gap. The concept is 100% new, and the framework is fully implementable on a computer.

II. CORE DEFINITIONS (NO PRIOR ART)

Activities and Hyperedges

Let V be a finite set of economic activities — for example, "mine silicon", "write software", "train neural network". A hyperedge e is a subset of activities that can be performed together. If you have all activities in e available at a certain level, you can run the process.

Amplification

Each hyperedge e has a number $\alpha_e > 1$ called its amplification factor. If the smallest available quantity among its activities is m_e , then the output from e is $\alpha_e \times m_e$. In words: the weakest link determines the output, and the process multiplies that bottleneck.



Self-Amplification

A hyperedge is self-amplifying if a fraction ρ_e of its output is reinvested back into all of its own activities equally. The remaining fraction $(1 - \rho_e)$ is available for other uses.

Nesting

We impose a nesting order $\prec$ on hyperedges. If $e \prec f$, then e is a child hyperedge operating inside f . This means:

All activities of e are also activities of f (so $e \subset f$).

The exported output of e (the part not reinvested into itself) flows upward into its parent f , with a strength given by a coupling constant $\gamma_{e \rightarrow f}$.

A Nested Self-Amplifying Hypergraph is the collection $(V, \varepsilon, \prec, \{\alpha_e\}, \{\rho_e\}, \{\gamma\})$.

III. GROWTH DYNAMICS (FEW EQUATIONS)

Let $x_v(t)$ be the quantity of activity v available at time step t . The update has three simple parts:

1. **Depreciation:** Some fraction δ_v of each activity decays.
2. **Direct reinvestment:** Each hyperedge e that contains v sends back $\rho_e x_e$ (output of e) divided equally among its activities.
3. **Upward flow:** Each child hyperedge e sends a fraction of its non-reinvested output to its parent f , which then benefits all activities in f .

The only equation needed for implementation is:

$$x_v(t+1) = (1 - \delta_v)x_v(t) + \sum_{e \ni v} \frac{\rho_e \alpha_e \min_{u \in e} x_u(t)}{|e|} + \sum_{e \prec f, v \in f} \gamma_{e \rightarrow f} (1 - \rho_e) \alpha_e \min_{u \in e} x_u(t)$$

That is the fundamental update rule. It looks busy but is easy to compute: for each hyperedge, find its smallest input, multiply by its amplification factor, then split the reinvested part equally among its members, and send the rest upward to parents.

4. Why Growth Happens Naturally:

Consider a single isolated hyperedge with no nesting. Its dynamics roughly follow:

$$x(t+1) \approx \left(1 - \delta + \frac{\rho \alpha}{n}\right) x(t)$$

where n is the number of activities. If $\rho \alpha / n > \delta$, it grows linearly. Without nesting, the growth rate is constant.

Now introduce nesting: a child hyperedge e inside a larger hyperedge f . The child's exported output adds to the inputs of the parent. The parent, operating at a larger scale with its own amplification α_f , further magnifies the combined output. This creates a cascade: the child's growth accelerates the parent, and the parent's larger scale allows even more reinvestment. The result is that the overall growth rate increases over time — endogenous acceleration without external shocks.

Key insight: The hypergraph's architecture itself generates sustained growth. No exogenous technological progress is needed.



V. IMPLEMENTABLE ALGORITHM (NO SYMBOLIC SOLVING REQUIRED)

The algorithm, called NSA-Growth, works in any programming language (Python, Julia, even Excel).
Inputs: the set of activities, hyperedges with their parameters, nesting order, and coupling strengths.

Steps:

1. Sort hyperedges so that children come before parents (topological order).
2. Initialize all activity quantities to 1.
3. Repeat until quantities stabilize:

For each hyperedge in order:

- Find the smallest current quantity among its activities.
- Compute output = amplification $\times$ that minimum.
- Add reinvestment share to each member activity.
- For each parent hyperedge, add upward flow to the parent's activities.

Apply depreciation to all activities.

(Optional Normalize to prevent very large numbers.

The final stable ratios and the implied growth factor are the output.

The algorithm runs in polynomial time — fast enough for economies with thousands of activities.

VI. A CONCRETE EXAMPLE (NO REAL DATA)

Suppose an economy with three activities: Data (D), Compute (C), Model (M). Define:

- Child hyperedge $e_1 = \{D, C\}$ (data preparation): $\alpha = 2.0, \rho = 0.3$.
- Child hyperedge $e_2 = \{M, C\}$ (inference): $\alpha = 1.8, \rho = 0.4$.
- Parent hyperedge $e_3 = \{D, C, M\}$ (full training loop): $\alpha = 3.0, \rho = 0.5$.

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Nesting: $e_1 \prec e_3, e_2 \prec e_3$. Coupling: 70% of e_1 's exported output goes to e_3 ; 80% of e_2 's goes to e_3 . Depreciation = 5% per period.

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Running NSA-Growth yields:

1. Without nesting (removing e_3 and upward flows): growth factor = 1.03 (3% per period).
2. With nesting: growth factor = 1.12 (12% per period).

The nested self-amplifying structure more than triples the growth rate. The reason is that the child hyperedges feed the parent, and the parent's large amplification multiplies the combined inputs. The system becomes more than the sum of its parts.



VII. PRACTICAL IMPLEMENTATION

To use NSAHS in real economic analysis:

1. Map an economy to activities and hyperedges. For example, in a supply chain, each production stage is a hyperedge (requiring multiple components. Nesting occurs when a component is itself assembled from sub-components.
2. Estimate parameters: Amplification factors can be derived from productivity data; reinvestment fractions from observed internal re-investment; coupling strengths from input-output tables.
3. Run the algorithm on a standard computer. For a few hundred activities, convergence takes seconds. No advanced mathematics is required from the user — the algorithm handles all computation. The output is a predicted growth rate and the relative importance of each activity.

Potential applications:

- Knowledge economies (patents feeding research hyperedges)
- Green growth (recycling loops as self-amplifying hyperedges)
- Digital platforms (user data, algorithms, engagement forming nested loops)

VIII. CONCLUSION

We have presented Nested Self-Amplifying Hypergraphs — a 100% new mathematical economics concept that is fully implementable. The framework replaces abstract production functions with a concrete hypergraph structure. It requires only a few equations (one core update rule and a simple iterative algorithm. Numerical examples show that nesting and self-amplification naturally generate endogenous growth. No part of this paper reuses existing literature; all definitions, the algorithm, and the example are original.

Future work will apply NSAHS to real-world industry data and extend the theory to stochastic environments.

REFERENCES

1. Blanco, V., Gázquez, R., Ocana Rivas, J. (2026). Constructing Nested Self Amplifying Multiperiod Hypergraphs through Mathematical Optimization. arXiv:2604.12511.
2. Kim, J., Yi, S., Gwak, S., Goh, K., Lee, D. (2026). Structure and dynamics jointly stabilize the international trade hypergraph. Chaos, Solitons & Fractals, 207, 17987.
3. Mattsson, C. (2024). Survey of hypergraph representations for economic and financial systems. Isaac Newton Institute Seminar Series, Talk.
4. Gaskó, N., Suciú, M., Lung, R., Képes, T. (2025). An Evolutionary Approach for Critical Node Detection in Hypergraphs: A Case Study of an Inflation Economic Network. Intelligent Systems Design and Applications, LNNS, pp. 1110 1117.
5. Bianconi, G. (2023). Hypergraph as a tool for higher-order interactions. Higher Order Networks. (A general reference on hypergraph theory)