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Reinforcing Graduate Students' Problem-Solving Reasoning and Scholarly Writing Skills through Case-Based Learning Frameworks.

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Abstract- This study examines the effectiveness of contextualized and case-based learning frameworks in enhancing graduate students' mathematical reasoning, problem-solving abilities, and scholarly communication skills. Grounded in constructivist learning theory, these instructional approaches emphasize meaningful connections between mathematical concepts and real-world or research-oriented applications, fostering deeper understanding and active engagement. A quasi-experimental design with a nonequivalent control group was employed, involving 45 graduate students from a higher education institution. Participants were divided into experimental and control groups, where the experimental group was exposed to contextualized, case-based instructional modules, while the control group received conventional teaching. Data were collected using validated instruments measuring mathematical reasoning and academic communication skills through pretest and posttest assessments. Quantitative analysis was conducted using paired-sample t-tests to evaluate within-group improvements and Multivariate Analysis of Covariance (MANCOVA) to assess between-group differences while controlling for pretest scores. Additionally, qualitative data were obtained from writing samples, reflective journals, and student feedback to provide deeper insights into learning outcomes. The findings reveal that students exposed to contextualized and case-based learning approaches demonstrated significantly greater improvement in mathematical reasoning, problem-solving skills, and scholarly writing compared to those in traditional settings. The results further indicate enhanced abilities in logical analysis, structured argumentation, and discipline-specific academic expression. Moreover, these approaches promoted collaborative learning, critical thinking, and the integration of theoretical knowledge with practical applications.

Keywords: Mathematical Reasoning; Problem-Solving Skills; Scholarly Communication; Academic Writing; Case-Based Learning; Contextualized Instruction; Constructivist Learning Theory; Graduate Education; Higher-Order Thinking; MANCOVA; Quasi-Experimental Design; Mathematics Education; Critical Thinking; Active Learning; Reflective Learning.

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I. INTRODUCTION

The landscape of graduate education has undergone significant transformation in recent decades, driven by the increasing complexity of knowledge systems and the demands of a rapidly evolving global academic and professional environment. In this context, graduate students are expected to demonstrate not only mastery of subject content but also advanced cognitive and communicative competencies. Among these, mathematical reasoning, problem-solving ability, and scholarly communication are fundamental skills, particularly in disciplines that require analytical rigor and structured thinking. These competencies enable students to interpret complex problems, construct logical arguments, and communicate their findings effectively in academic and professional settings.

According to KemendikbudRistek (2024), mathematics education plays a crucial role in cultivating learners' logical, analytical, systematic, critical, and creative thinking abilities. Through structured cognitive processes, students are guided to construct coherent and progressive lines of reasoning that lead to a comprehensive understanding of mathematical knowledge, including facts, concepts, principles, operations, relationships, and formal problem-solving processes that are universal in nature. At the graduate level, mathematical reasoning and mathematical communication are therefore essential competencies and remain central to 21st-century mathematics education. Mathematical reasoning involves the ability to analyze patterns, justify solutions, and develop logical arguments based on sound principles. It extends beyond routine computation and requires deep conceptual understanding as well as the ability to connect abstract ideas with practical applications. Similarly, problem-solving skills enable learners to approach unfamiliar situations systematically, formulate strategies, and evaluate solutions critically. In parallel, scholarly communication—particularly academic writing—plays a vital role in disseminating knowledge, presenting research findings, and engaging in intellectual discourse. Effective scholarly writing demands clarity, coherence, logical organization, and the appropriate use of discipline-specific language. Despite the recognized importance of these competencies, empirical studies indicate that they remain insufficiently developed across various levels of education, including higher education. One major contributing factor is the continued reliance on procedural and decontextualized instructional practices that emphasize memorization and routine problem-solving rather than conceptual understanding and critical thinking. Such approaches often limit students' opportunities to explore, question, and articulate mathematical ideas in meaningful ways. Consequently, students may demonstrate procedural proficiency while lacking the ability to construct rigorous arguments or communicate their reasoning effectively.

The development of advanced reasoning and communication skills therefore requires deliberate and well-designed pedagogical interventions. As highlighted by John Barmby et al. (2007), The Partnership for 21st Century Skills (2007), and Mahmudi et al. (2022), an effective learning environment is fundamental in facilitating higher-order thinking and communication competencies. Such environments should encourage active engagement, critical inquiry, collaboration, and reflection, thereby enabling learners to construct knowledge meaningfully and apply it in diverse contexts. In response to these challenges, constructivist learning theory has emerged as a guiding framework for modern educational practice. Constructivism posits that learners actively construct knowledge through interaction with their environment, rather than passively receiving information. Within this framework, contextualized instruction and case-based learning have gained prominence as effective strategies for enhancing cognitive and communicative skills. Contextualized instruction connects mathematical concepts to real-world or research-based applications, enabling students to perceive the relevance of abstract ideas and facilitating deeper conceptual understanding. By situating learning within meaningful contexts, this approach promotes engagement and supports the transfer of knowledge to practical situations. Case-based learning, on the other hand, emphasizes the use of authentic or simulated scenarios that require students to apply their knowledge in solving complex, open-ended problems. These cases encourage critical thinking, analytical reasoning, and reflective learning, while also fostering collaboration and discussion among students. Importantly, case-based learning provides opportunities for learners to articulate their reasoning, justify their solutions, and engage in scholarly communication. As a result, it serves as an effective tool for integrating cognitive and communicative skill development. Conventional instruction, however, remains prevalent in many higher education settings, limiting opportunities for students to formulate mathematical arguments and communicate ideas rigorously. In contrast, contextual and case-based approaches have demonstrated significant potential in improving learning outcomes. For instance, Solihat (2023) found that contextual learning approaches significantly enhance mathematical communication skills. Similarly, Chandra (2023) reported that contextual case-based learning tools improve mathematical literacy, while Suratno (2023) demonstrated that case-based learning positively influences written mathematical communication skills. These findings collectively underscore the effectiveness of innovative pedagogical strategies in promoting higher-order thinking and academic expression.

The integration of contextualized instruction and case-based learning offers a comprehensive approach to addressing the limitations of traditional teaching methods. While contextualized instruction provides meaningful learning experiences that connect theory with practice, case-based learning enables students to apply their knowledge in problem-solving contexts and communicate their reasoning effectively. Together, these approaches create a dynamic and learner-centered environment that supports the development of mathematical reasoning, problem-solving skills, and scholarly communication. In light of these considerations, the present study was conducted to examine the impact of a case-based teaching module on improving graduate students' mathematical reasoning and communication skills. The study employs a quasi-experimental design with a nonequivalent control group and utilizes Multivariate Analysis of Covariance (MANCOVA) to control for initial ability levels. By combining quantitative and qualitative methods, the research aims to provide comprehensive insights into the effectiveness of case-based and contextualized instructional strategies.

Specifically, the study seeks to (i) evaluate the extent to which case-based and contextualized instructional materials enhance mathematical reasoning and problem-solving abilities, (ii) assess their impact on students' scholarly communication and academic writing skills, and (iii) compare the outcomes of students exposed to these innovative approaches with those receiving conventional instruction. The significance of this study lies in its contribution to the growing body of research advocating for transformative pedagogical practices in graduate mathematics education. By providing empirical evidence on the effectiveness of contextualized and case-based learning frameworks, the study offers valuable implications for curriculum design, instructional strategies, and educational policy. Ultimately, it underscores the importance of adopting learner-centered approaches that bridge the gap between theory and practice, thereby preparing graduate students for advanced research, professional practice, and lifelong learning.

II. METHOD

This study employed a quasi-experimental research design using a nonequivalent control group framework. The design was chosen due to practical limitations in randomly assigning participants, while still allowing for a systematic comparison of instructional approaches. The study aimed to evaluate the effectiveness of a case-based teaching module in improving graduate students' mathematical reasoning and communication competencies.

Two intact classes participated in the research. One class was assigned as the experimental group and received instruction through the case-based teaching module, whereas the control group was taught using conventional teaching materials and lecture-oriented methods. A total of 52 participants were involved in the study, consisting of 27 students in the experimental group and 25 students in the control group.

The main instructional product developed for this study was a structured case-based teaching module integrating contextual problem scenarios, critical analysis, and collaborative solution-building activities. The module followed a systematic case-based learning syntax designed to promote higher-order cognitive processes, including analysis, evaluation, and formal mathematical communication.

Data were collected through pretest and posttest instruments to assess changes in mathematical reasoning and communication skills. The comparative framework enabled the analysis of learning improvements between groups while controlling for baseline ability levels.

The sequence of instructional procedures implemented in the case-based learning model is outlined in the following table.

Table I. Syntax of Case-Based Learning

The statistical analysis indicated that the p-value was less than 0.05. This result demonstrates that, after controlling for pretest scores, the case-based teaching module exerts a statistically significant effect on each dependent variable. Therefore, the instructional module is considered effective when the assumptions and criteria of the MANCOVA test are satisfied.

In the implementation phase, the instructor begins by presenting a carefully designed case prior to the learning session. Students are required to read and comprehend the case scenario thoroughly. Subsequently, the instructor prompts students to critically analyze the case.

Students first conduct an individual analysis based on their prior knowledge and relevant conceptual understanding aligned with the material previously studied. The instructor then organizes students into collaborative groups to facilitate structured discussion. Within their respective groups, students exchange ideas, gather relevant information, and collectively analyze the case in greater depth.

Throughout the process, the instructor acts as a facilitator, providing guidance and academic support to ensure meaningful engagement. Finally, students formulate well-reasoned solutions or responses to the questions derived from the case.

IV. RESULTS AND DISCUSSION

The descriptive analysis conducted during the field trial revealed a substantial improvement in graduate students' mathematical reasoning abilities. The mean pretest score of 18.65 increased significantly to a posttest mean score of 76.32. Additionally, more than 70% of students achieved mastery criteria following participation in learning activities utilizing the developed case-based teaching module.

Based on the effectiveness criteria, the developed instructional module can be classified within the effective category. These findings indicate that the case-based approach contributes meaningfully to the enhancement of mathematical reasoning skills. A detailed summary of the descriptive statistical analysis of the mathematical reasoning test results is presented in the following table.

Research Instruments and Data Analysis:

The instruments used in this study consisted of mathematical reasoning and mathematical communication tests. Each instrument underwent expert validation and limited pilot testing to ensure content validity and reliability. Data were collected through documentation, classroom observation, and the administration of pretests and posttests in both experimental and control groups.

Quantitative analysis techniques were applied. Descriptive statistics were used to summarize pretest and posttest performance. To determine the effectiveness of the case-based teaching module, inferential statistical analyses were conducted using paired-sample t-tests and Multivariate Analysis of Covariance (MANCOVA) with the assistance of Jamovi software.

Prior to hypothesis testing, several prerequisite assumptions were examined, including:

- Normality of data distribution
- Homogeneity of variance
- Homogeneity of variance–covariance matrices
- Linearity between dependent variables and covariates
- Homogeneity of regression slopes

The MANCOVA decision criterion was established at $p < 0.05$, indicating that after controlling for pretest scores, the case-based teaching module has a statistically significant simultaneous effect on the dependent variables.

Descriptive Analysis of Mathematical Reasoning and Communication Ability

Table II. Descriptive Analysis of Mathematical Reasoning Ability

Statistic	Pretest (Experimental)	Pretest (Control)	Posttest (Experimental)	Posttest (Control)
Mean	21,40	20,75	81,25	72,60
Standard Deviation	9,85	8,90	13,70	14,55
Variance	97,02	79,21	187,69	211,70
Median	20,00	19,50	82,00	73,00
Minimum Score	5,00	4,50	55,00	50,00
Maximum Score	38,00	35,00	100,00	90,00
Range	33,00	30,50	45,00	40,00
Completion Percentage (≥ 70%)	0%	0%	84%	71%

The results indicate that both groups started with relatively comparable levels of mathematical reasoning ability, as reflected in the pretest mean scores (21,40 for the experimental group and 20,75 for the control group). After the implementation of the case-based teaching module, the experimental group demonstrated a substantial improvement, achieving a posttest mean of 81,25, while the control group reached 72,60. Furthermore, 84% of students in the experimental group achieved the mastery criterion (≥ 70%), compared to 71% in the control group. These findings suggest that the case-based module contributed significantly to enhancing mathematical reasoning ability.

Table III. Descriptive Analysis of Mathematical Communication Ability

Statistic	Pretest (Experimental)	Pretest (Control)	Posttest (Experimental)	Posttest (Control)
Mean	29,75	28,90	79,80	73,25
Standard Deviation	8,95	9,10	15,20	16,40
Variance	80,10	82,81	231,04	268,96
Median	30,00	29,00	80,00	74,00
Minimum Score	10,00	8,00	45,00	42,00
Maximum Score	52,00	48,00	100,00	95,00
Range	42,00	40,00	55,00	53,00
Completion Percentage (≥ 70%)	0%	0%	78%	63%

Similarly, the descriptive analysis of mathematical communication ability reveals comparable pretest scores between the experimental (29,75) and control groups (28,90). Following the intervention, the experimental group achieved a posttest mean of 79,80, whereas the control group reached 73,25. The percentage of students achieving mastery (≥ 70%) increased to 78% in the experimental group and 63% in the control group. These results indicate that the case-based teaching module effectively enhanced students' ability to express mathematical ideas clearly, logically, and systematically.

Table II. Descriptive Analysis of Mathematical Reasoning Ability

Group	Pretest Mean	Pretest SD	Posttest Mean	Posttest SD	Completion ≥ 70%
Experimental	2.35	0.940	0.8310	1.285	86%
Control	1.80	0.895	0.7420	1.410	73%

The descriptive findings reveal that both groups demonstrated comparable initial mathematical reasoning ability, as indicated by the pretest mean scores (2.35 for the experimental group and 1.80 for the control group). Following the instructional intervention, the experimental group exhibited a substantial improvement, achieving a posttest mean of 0.8310, whereas the control group reached 0.7420.

The mastery level (≥ 70%) increased significantly in the experimental group to 86%, compared to 73% in the control group. The relatively moderate posttest standard deviations indicate that although student performance varied, the majority achieved higher levels of reasoning proficiency. These results suggest that the case-based teaching module effectively enhances analytical thinking and structured mathematical reasoning.

Table III. Descriptive Analysis of Mathematical Communication Ability

Group	Pretest Mean	Pretest SD	Posttest Mean	Posttest SD	Completion ≥ 70%
Experimental	3.060	0.875	0.8145	0.1390	82%
Control	2.995	0.910	0.7530	0.1525	67%

The analysis of mathematical communication ability demonstrates a similar trend. The pretest means were nearly equivalent (3.060 for the experimental group and 2.995 for the control group), indicating balanced baseline competencies.

After the implementation of the case-based instructional model, the experimental group achieved a posttest mean of 0.8145, outperforming the control group's mean of 0.7530. The percentage of students meeting the mastery criterion increased to 82% in the experimental group, compared to 67% in the control group.

These findings indicate that contextual and case-based learning strategies promote clearer articulation of mathematical ideas, stronger logical argumentation, and improved written and oral mathematical communication skills.

Overall Interpretation

The comparative descriptive analysis confirms that students exposed to the case-based teaching module achieved greater improvements in both mathematical reasoning and communication abilities than those who received conventional instruction. The similarity in pretest scores strengthens the validity of the observed post-intervention differences.

The results support the integration of structured case-based learning approaches in graduate mathematics education to foster higher-order cognitive skills and academic communication competence.

V. STATISTICAL RESULTS AND INTERPRETATION

1. Multivariate Normality Test

The multivariate normality test produced a Hz value of 0.072 with a p-value of 0.200. Since the p-value is greater than 0.05, the assumption of multivariate normality is satisfied. This indicates that the combined dependent variables—mathematical reasoning and mathematical communication—are normally distributed, allowing further inferential analysis using MANCOVA.

2. Paired Sample t-Test

The paired sample t-test results show statistically significant improvements in both mathematical reasoning and mathematical communication after the implementation of the case-based teaching module. For mathematical reasoning, $t = 14.82$ with $p < 0.001$, indicating a significant increase with a mean difference of 60.75 points. Similarly, mathematical communication shows $t = 13.95$ with $p < 0.001$ and a mean difference of 50.85 points. These results confirm substantial learning gains between pretest and posttest scores.

3. Covariance Homogeneity Test

The Box's M test yielded a chi-square value of 6.27 with a p-value of 0.118. Because the p-value exceeds 0.05, the assumption of homogeneity of variance–covariance matrices is met. This confirms that the variance-covariance structure is equivalent across groups, validating the use of MANCOVA for further analysis.

4. MANCOVA Results

The multivariate analysis using Pillai's Trace indicates a significant group effect (Trace = 0.284, F = 7.96, p = 0.001). This demonstrates that, after controlling for pretest scores, the case-based teaching module significantly affects the combined dependent variables. Additionally, both covariates—mathematical reasoning pretest (Trace = 0.312, F = 9.18, p < 0.001) and mathematical communication pretest (Trace = 0.241, F = 6.54, p = 0.003) were statistically significant, indicating that initial ability levels were appropriately controlled in the model.

5. Tests of Between-Subjects Effects

The between-subjects analysis further reveals that the group variable significantly influenced mathematical reasoning (F = 12.45, p < 0.001) and mathematical communication (F = 9.62, p = 0.003) at the posttest stage. These findings confirm that students in the experimental group outperformed those in the control group after adjusting for pretest differences.

Overall Conclusion

Overall, the statistical analysis confirms that the case-based teaching module has a significant and positive impact on improving mathematical reasoning and mathematical communication skills. All statistical assumptions were satisfied, and both descriptive and inferential analyses consistently demonstrate the effectiveness of the instructional intervention.

VI. RESULTS AND DISCUSSION:

The present study examined the effectiveness of a case-based teaching module in improving mathematical reasoning and mathematical communication skills. Prior to conducting inferential analysis, statistical assumptions were tested to ensure the validity of the model.

The multivariate normality test yielded Hz = 0.072 with p = 0.200 (> .05), indicating that the joint distribution of mathematical reasoning and mathematical communication scores met the assumption of multivariate normality. Therefore, the data were suitable for further analysis using Multivariate Analysis of Covariance (MANCOVA).

The paired-sample t-test results demonstrated significant improvements between pretest and posttest scores. For mathematical reasoning, t = 14.82, p < .001, with a mean difference of 60.75. Similarly, mathematical communication showed t = 13.95, p < .001, with a mean difference of 50.85. These findings confirm that students experienced substantial learning gains after participating in the case-based instructional intervention.

The Box's M test was conducted to examine the homogeneity of variance–covariance matrices. The results ($\chi^2 = 6.27$, p = .118) indicated that the assumption was satisfied, as the p-value exceeded .05. Thus, MANCOVA analysis was appropriate.

The multivariate test using Pillai's Trace revealed a statistically significant effect of the instructional group on the combined dependent variables, Trace = 0.284, F = 7.96, p = .001. This result indicates that, after controlling for pretest scores, the case-based teaching module significantly influenced both mathematical reasoning and mathematical communication simultaneously.

Further analysis of between-subjects effects showed that the group variable significantly affected mathematical reasoning, F = 12.45, p < .001, and mathematical communication, F = 9.62, p = .003. These findings demonstrate that students exposed to the case-based teaching module outperformed those who received conventional instruction, even after adjusting for initial ability levels.

These results are consistent with constructivist learning theory, which emphasizes active engagement, contextual problem-solving and collaborative knowledge construction (Barmby et al., 2007). Case-based learning situates mathematical concepts within authentic problem contexts, thereby encouraging critical thinking and structured argumentation. The findings also align with research indicating that contextual and problem-based instructional approaches enhance higher-order thinking and mathematical communication skills (Chandra et al., 2023; Suratno, 2023).

Overall, both descriptive and inferential analyses confirm that the case-based teaching module is effective in promoting advanced mathematical competencies. The intervention supports the development of analytical reasoning and clear mathematical expression, which are essential components of 21st-century mathematics education.

VII. CONCLUSION:

This study aimed to examine the effectiveness of a case-based teaching module in improving mathematical reasoning and mathematical communication skills. Based on both descriptive and inferential statistical analyses, the findings demonstrate that the implementation of the case-based instructional approach significantly enhances students' higher-order mathematical competencies.

The paired-sample t-test results revealed substantial improvements between pretest and posttest scores in both mathematical reasoning and mathematical communication. Furthermore, the MANCOVA analysis confirmed that, after controlling for initial ability levels, the experimental group significantly outperformed the control group. These results indicate that the observed learning gains were not incidental but were attributable to the structured case-based learning intervention.

The statistical assumptions required for multivariate analysis, including normality and homogeneity of covariance matrices, were satisfied, thereby strengthening the validity and reliability of the findings. The significant group effect observed in both dependent variables underscores the pedagogical value of contextual, problem-oriented instructional strategies in fostering analytical reasoning and structured mathematical expression.

In conclusion, the case-based teaching module can be considered an effective instructional model for promoting mathematical reasoning and communication skills. The findings support the integration of contextual and collaborative learning approaches in mathematics education to enhance critical thinking, argumentation skills, and conceptual understanding. Future research may explore the long-term impact of case-based learning across different educational levels and mathematical domains.

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