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A Study of the Double Soham Transform and its Utility in Applied Sciences

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Abstract- In this article, we examine a new double transform which is a com- bination of ARA transformand Sawi transform (double ARA-SW). We present some basic properties of double ARA-SW transform like linearity, shifting, ex- istance and uniqueness and double convolution theorem. We proved some important results of double ARA-SW transform related to partial derivatives. In order to show that useablity of double ARA-SW transform, some examples of partial differential equation are illustrated.

Keywords- Double ARA-SW Transform, ARA Transform, Sawi Transform, Linearity, Shifting, Existence and Uniqueness

I. INTRODUCTION

Integral transforms are essential tools in mathematics, providing a way to sim- plify complex functions for easier analysis and problem-solving. By transforming a problem from one domain (such as time or space) into another (like frequency or spectral domain), these methods allow for more manageable computations, es- pecially when dealing with differential, integral, and partial differential equations. Classical transforms like the Laplace and Fourier transforms have been widely used in various fields of science and engineering for many years [1,2]. Recently, how- ever, new variations and combinations of these transforms have been developed to handle more complicated problems. These include the Double ARA-Sumudu trans- form [3], the Laplace-ARA transform [4], and even the Triple Shehu transform [5]. These newer methods expand the toolbox for solving advanced issues like fractional differential equations, boundary value problems, and integral equations [4,6]. The value of these transforms is clear—they provide practical solutions to mathematical problems that might otherwise be intractable in their original forms. Whether in physical sciences, engineering, or applied mathematics, they offer efficient ways to analyze complex systems.

One such innovative transform is the Soham transform, which has shown promise in solving various integral equations and modeling real-world phenomena [7,8]. Building on this, the Double Soham Transform is introduced as a new extension. This new transform offers a powerful framework for tackling boundary value prob- lems and other mathematical challenges. This paper explores the theory behind the Double Soham Transform and showcases its practical applications. By extending the principles of existing double integral transforms like the Double ARA-Sumudu and Double Sawi transforms [3,9], we demonstrate how the Double Soham Trans- form can become a valuable tool for solving a wide range of problems more effec- tively.

Key words and phrases. ARA transform, Sawi transform, double ARA-Sawi transform and partial differential equation.

The paper is organized in a way that builds up the concept step by step. It starts with Section 2, where the basics of the Soham transform are explained, covering its definitions and properties. Then, in Section 3, the author introduces a new idea: the Double Soham Transform. Section 4 takes this further by showing how the Double Soham Transform can be applied to various functions. In

Section 5, the paper dives into proving some key properties of the Double Soham Transform. Moving on to Section 6, the author applies this transform to solve partial differ- ential equations (PDEs), with a few practical examples. Finally, Section 7 wraps things up with a conclusion, summarizing the key points and findings.

Preliminaries

Laplace transform. Let μ : (0, ∞) → R represent a real-valued function. The single Laplace transform of μ can be written as:

∫ ∞

μ(s) = L[μ(τ)] = e−sτ μ(τ) dτ, s ∈ C. (2.1)

0

Double Laplace transform. Let μ : (0, ∞) → R represent a real-valued function. The single Laplace transform of μ(τ, y) can be written as:

∫ ∞ ∫ ∞

Lτ Ly[μ(τ, y)] = μ(s, p) = e−()sτ+py)μ(τ, y) dτdy, (s, p) ∈ C. (2.2)

0 0

Shehu transform. The single Shehu transforms (ST) of a real-valued func- tion f (q, t), with respect to the variables q and t, are described by the following definitions:

Sq (f (q, t)) = F (h, m) =

∞

e−(hq)

0

f (q, t)dq (2.3)

St (f (q, t)) = F (i, n) =

∞

e−(it)

0

f (q, t)dt. (2.4)

Double Shehu transform. The double Shehu transform (DST) of the func- tion f (q, t) is described by,

2 ∫ ∞ ∫ ∞ −(hq + it)

Sqt[f (q, t)] = F [(h, i), (m, n)] = 0

2.5. Soham transform (S).

e m n

0

f (q, t)dqdt. (2.5)

Definition 2.1. The Soham transform operates on a well-defined set of functions

g(y), y ≥ 0

D = {g(y) : ∃M, ν1, ν2 > 0 such that g(y) < Me|y|kj , if y ∈ (−1)j × (0, ∞)}

using the provided formula,

S [g(y)] = ∫ g(y)e−u ydy = G(u) (2.6)

1 ∞

u 0

The Soham transform, along with its inverse, is described using the following for- mulas. These definitions apply for u ≥ 0

S −1[G(u)] = g(y) = 1 ∫ β+i∞ 1 eu yG(u)du. (2.7)

2πi β−i∞ u

Remark 2.2. If α = 1,

S [g(y)] = 1 ∫ ∞

g(y)e−uydy = G(u) y ≥ 0, (2.8)

u 0

this is the definition of Aboodh transform.

Remark 2.3. If α = −1 then,

1 ∫ ∞ 1

S [g(y)] = g(y)e− u ydy = G(u) y ≥ 0, (2.9) 0

this is the definition of Sumudu transform.

Remark 2.4. If α = −2 then,

S [g(y)] = 1 ∫ ∞

g(y)e

— 1 y u

dy = G(u) y ≥ 0,(2.10)

u 0

this is the definition of New integral transform.

Double Soham Transform (DST)

This section describes the definition for double Soham transform and its inverse.

Definition 3.1. The double Soham transform of the function g(y, t) can be ex- pressed as follows:

S 2 [g(y, t)] = G(u, v) = 1 ∫ ∞ ∫ ∞ e−u y−vβtg(y, t)dydt, u ≥ 0, v ≥ 0, (3.1)

yt uv 0 0

provided the integral converges, the inverse of double Soham transform is expressed as:

S −2[G(u, v)] = g(y, t) = 1 ∫ ω+i∞ ∫ λ+i∞ 1 eu y+v G(u, v)dudv.(3.2)

uv 2πi ω−i∞ λ−i∞ uv

Double Soham Transforms For Some Fundamental Functions

(1) Let g(y, t) = 1 then
S 2 [1] = 1 .
yt (uv)α+1
(2) Let g(y, t) = yntm, n, m = 0, 1, 2... then
S 2 [yntm] = 1
n! m!
yt uv u(n+1)αv(m+1)β
(3) Let g(y, t) = yα1 tα2 , α1 ≥ −1, α2 ≥ −1 then
S 2 [yα1 tα2] = 1 Γ(α1 + 1)Γ(α2 + 1) ,
yt uv u(α1+1)αv(α2+1)β
where Γ. is the Euler gamma function.
(4) Let g(y, t) = eay+bt then
S 2 [eay+bt] = 11
yt
Consequently
uv (uα − a)(vβ − b)
S 2 ei(ay+bt) = 1 1
2 uαb + vβa
Syt[sin(ay + bt)] = uv(u2α + a2)(v2β + b2)
2 uαvβ − ab
Syt[cos(ay + bt)] = uv(u2α + a2)(v2β + b2)
S 2 [sinh(ay + bt)] = 1 1 − 1
S 2 [cosh(ay + bt)] = 1 1 + 1 .

Existence and uniqueness of double Soham transform.

Theorem 4.1. Let g(y, t) be a continuous function defined on any finite intervals
(0, Y) and (0, T), and of exponential order, implying that for some a, b ∈ R
|g(y, t)|
sup
y,t>0
eay+bt < ∞,
then the double Soham transform of g(y, t) exists.
Proof. By applying the definition of double Soham transform,
1 ∫ ∞ ∫ ∞ α β
S 2 [g(y, t)] =
e−u
y−v tg(y, t) dy dt
yt uv 0 0
1 ∫ ∞ ∫ ∞ e−u y−vβt|g(y, t)|dydt
uv 0 0
M ∫ ∞ ∫ ∞ e−u y−vβ teat+btdydt
uv 0 0
≤ M e−(uα
∞
−a)ydy
e−(vβ −b)tdt
uv 0 0
M
= uv(uα − a)(vβ − b)

Theorem 4.2. Let g(y, t) and f (y, t) be a continuous functions and having the dou- ble Soham transform S 2 [g(y, t)] and S 2 [f (y, t)] resp. If S 2 [g(y, t)] = S 2 [f (y, t)]
yt yt
then g(y, t) = f (y, t).
yt yt
Proof. Assume α1and α2 are sufficiently large, then
g(y, t) = S −2[S 2 g(y, t)] = 1 ∫ α1+i∞ eu ydu 1 ∫ α2+i∞ evβtdv
We conclude that, yt
1 ∫ α1+i∞
2πi α
α1−i∞
1 ∫ α2+i∞
2πi
β
α2−i∞
g(y, t) = eu ydu ev tS 2 g(y, t)dv
2πi α1−i∞ 2πi α2−i∞
1 α1+i∞
= eu ydu
1 ∫ α2+i∞
evβtS 2 f (y, t)dv
2πi
α1−i∞
2πi
yt
α2−i∞
= f (y, t)

Basic Properties of Double Ara-Sw Transform

Shifting property. If the double double ARA-Sawi transform of function
g(x, y) is a function G(s, u) then for any pair of real constants α, β > 0
A S eαx+βyg(x, y) = s G s − α, u

Proof. By using definition of double D double ARA-SW transform,

AxSy eαx+βyg(x, y) ∫ ∫ e−sx− +αx+βyg(x, y)dxdy

s ∞ ∞ y

u2 0 0

s ∞ ∞

= 2

e−[(s−α)x+(1 −β)y]

g(x, y)dxdy

u 0 0

A S eαx+βyg(x, y) = s G s − α, u

x y (s − α)(1 − βu)2

1 − βu

Linearity property. If the double ARA-SW transform of functions g1(x, y) and g2(x, y) are G1(s, u) and G2(s, u) respectively, then double ARA-SW transform of αg1(x, y) + βg2(x, y) is given by αG1(s, u) + βG2(s, u), where α, β are arbitrary constants.

AxSy[αg1(x, y) + βg2(x, y)] = αG1(s, u) + βG2(s, u)

proof:

s

AxSy[αg1(x, y) + βg2(x, y)] =∫ ∞ ∫ ∞

e−(sx+ y)[αg (x, y) + βg (x, y)]dxdy

s ∫ ∞ ∫ ∞

u2 0

y

0

s ∫ ∞ ∫ ∞ y

= 2 α

0

e−(sx+ u)g1(x, y)dxdy + β

0 u2 0

e−(sx+ u)g2(x, y)dxdy

0

s ∫ ∞ ∫ ∞y

AxSy[αg1(x, y) + βg2(x, y)] = e−(sx+)g (x, y)dxdy

u2 0 0

s ∞ ∞

+ β

e−(sx+ y)g (x, y)dxdy

u2 0 0

AxSy[αg1(x, y) + βg2(x, y)] = αG1(s, u) + βG2(s, u)

Change of scale. Let G(s, u) be the double ARA-SW transform of function

g(x, y), then for α and β are positive constants, we have

A S [g(αx, βy)] = 1 G s , βu

Proof: Using the definition of double ARA-SW transform we get, s

AxSy[g(αx, βy)] =

∫ ∞ ∫ ∞

e−(sx+ y)g(αx, βy)dxdy

u2 0 0

set ζ = αx, η = βy then

s ∫ ∞ ∫ ∞ (sζ + η)

AxSy[g(αx, βy)] = αβ

e α βu

0

g(ζ, η)dζdη

A S [g(αx, βy)] = 1 G s , βu

Derivative properties.

If S 2 [g(y, t)] = G(u, v) then

S 2 ∂g(y, t) = uαG(u, v) − 1 S [g(0, t)]

yt ∂y u

Proof:-

S 2 ∂g(x, y) = 1 ∫ ∞ ∫ ∞

e−u

y−vβ t

∂g(y, t)

dydt

yt ∂y

1 ∫ ∞

uv 0 0

β ∫ ∞

∂y

α ∂g(y, t)

Now,

= uv 0

e−v tdt

0

e−u y

dy (5.1)

∂y

∞

e−u y

0

∂g(y, t)

∂y

dy =

he−uα

yg(y, t)

∫ ∞
∞
+ uα
0
α
∞
e−u
0
g(y, t)dy
= −g(0, t) + uα
0
e−u
yg(y, t)dy
S 2 ∂g(y, t) =
1 ∫ ∞
evβtg(0, t)dt +
uα ∫ ∞ ∫ ∞
e−(u
y+vβt)g(y, t)dydt
yt ∂y
uv 0
uv 0 0
= uαG(u, v) − 1 S [g(0, t)] u

If S 2 [g(y, t)] = G(u, v) then
S 2 ∂g(y, t) = vβG(u, v) − 1 S [g(y, 0)]
Proof:- Proof of this property is same as proof of (5.4.1).
5.4.3. If S 2 [g(y, t)] = G(u, v) then
2 ∂2g(y, t)
−1 ∂G(0, t) 2α
(α−1)
Proof.
∂2g(y, t)
1 ∫ ∞ ∫ ∞
e−u
y−vβ t
∂2g(y, t)
dydt
yt ∂y2
1 ∫ ∞
uv 0 0
β ∫ ∞
∂y2
α ∂2g(y, t)
=
uv 0
e−v tdt
0
e−u y
∂y2 dy
1 ∞
= e−vβ tdt
e−uαy
∂g(y, t) ∞
∞
+ uα
e−u y
∂g(y, t)
dy
uv 0
1 ∫ ∞
∂y 0
β ∂g(0, t)
0 ∂y
1
= −
uv 0
e−v t
dt + uα
∂y
uαG(u, v) −
u S [g(0, t)]
2 ∂2g(y, t)
−1 ∂G(0, t) 2α
(α−1)
Syt
∂y2
= + u
u ∂y
G(u, v) − u

G(0, v).

If S 2 [g(y, t)] = G(u, v) then
2 ∂2g(y, t)
−1 ∂G(y, 0) 2β
(β−1)
Proof.
∂2g(y, t)
1 ∫ ∞ ∫ ∞
e−u
y−vβ t
∂2g(y, t)

dydt
yt ∂t2
1 ∞
= uv
e−u
0 0
∞
ydy
e−vβ t

∂t2
∂2g(y, t)
dt
1 ∫ ∞
uv 0
α
0
β ∂g(y, t) ∞
∂t2
∫ ∞
β ∂g(y, t)
=uv 0
e−u
ydy
e−v t
∂t
+ vβ
0 0
e−v t dt
∂t
1 ∞
= −
e−u y
∂g(y, 0)
dy + vβ
vβG(u, v) −
1 S [g(y, 0)]
uv 0 ∂t v
2 ∂2g(x, y)
−1 ∂G(y, 0) 2β
(β−1)
Syt
∂t2
= + v
v ∂t
G(u, v) − v
G(u, 0).

If S 2 [g(y, t)] = G(u, v) then
2 ∂2g(y, t)
−1 ∂G(0, t) 2α
(α−1)
Proof.
∂2g(y, t)
1 ∫ ∞ ∫ ∞
e−u
y−vβ t
∂2g(y, t)
dydt
yt ∂y∂t
1 ∞
= uv
e−u
0 0
∞
ydy
e−vβ t
∂y∂t
∂2g(y, t)
dt
1 ∫ ∞
uv 0
α
0
β ∂g(y, t) ∞
∂y∂t
∫ ∞
β ∂g(y, t)
= uv 0
e−u
ydy
e−v t
∂y
+ vβ
0 0
e−v t dt
∂y
1 ∞
= − e−u y
∂g(y, 0)
dy +
vβ ∫ ∞ ∫ ∞
e−u
y−vβ t
∂g(y, t)

dydt
uv 0 ∂y uv 0 0 ∂y
1 ∫ ∞ α
= − − g(0, 0) + uα
uv 0
e−u
yg(y, 0) dy
vβ ∞
+
e−vβ t dt
∞
— g(0, t) + uα
e−u
yg(y, t) dy .
uv
2 ∂2g(y, t)
0
−g(0, 0)
uα ∂G(y, 0)
0
vβ ∂G(0, t) α β
Syt
∂y∂t
= −
uv v
—
∂y u
+ u v
∂t
G(u, v

Applications

Example 6.1.
with the conditions,
∂g(y, t)
∂y
∂g(0, t)
= ∂t
(6.1)
g(y, 0) = y, g(0, t) = t. (6.2)
Solution:
1 1
S [g(y, 0)] = v2β+1 , S [g(0, t)] = u2α+1
vβG(u, v) − 1 S [g(0, t)] = uαG(u, v) − 1 S [g(y, 0)]
(6.3)
v
(vβ − uα)G(u, v) = 1
uv2β+1
vu2α+1 − uv2β+1
u
1
— vu2α+1
1
G(u, v) = ×
uvu2α+1v2β+1 (vβ − uα)
1 1

Example 6.2.
G(u, v) = uα+1v2β+1 + u2α+1vβ+1 (6.4)
g(y, t) = y + t
∂g(y, t) + ∂g(0, t) = et(1 + y) (6.5)
∂y ∂t
with the conditions,
Solution:
g(y, 0) = y, g(0, t) = 0. (6.6)
1
S [g(y, 0)] = v2β+1 ,
vβg(u, v)− 1 S g(0, t)+uαG(u, v)− 1 S g(y, 0) = 1
1 + 1 (6.7)
β α 1 vβ + 1 1

vβ + 1 1 1
G(u, v) = u(uα − 1)v2β+1 + uv2β+1
1
× (uα + vβ)

Example 6.3.
G(u, v) = u(uα − 1)v2β+1 (6.8)
g(y, t) = yet.
with initial conditions,
∂2g(y, t)
∂y2 +
∂2g(y, t)
∂t2 − 4g(y, t) = 0 (6.9)
and boundary conditions,
g(y, 0) = y, ∂g(y, 0) = e−2y (6.10)
∂t
∂g(0, t)
g(0, t) = t,
= −2t. (6.11)
∂y

Solution:
S [g(y, 0)] = 0, S ∂g(y, 0) = 1
S [g(0, t)] = 1 , S ∂g(0, t) = −2

v2βG(u, v) − vβ−1S [g(0, t)]−v−1S ∂g(0, t) +
u2αG(u, v) − uα−1S [g(y, 0)] ∂y
— u−1S∂g(y, 0) − 4G(u, v) = 0 (6.12)
∂t
vβ 2 1
1
G(u, v) =
vu2α+1 − v(u2α+1) + uv(vβ + 2)
1
u2α + v2β − 4
G(u, v) = vu2α+1(vβ + 2) (6.13)
g(y, t) = te−2y

II. CONCLUSIONS

The paper presents a new mathematical tool called the Double Soham Trans- form. It aims to make solving complex problems, like fractional differential equa- tions, easier. This transform builds on well-known methods like Laplace and Fourier transforms but offers a more powerful approach for tackling advanced equations. The paper explains how the Double Soham Transform works, proves its key prop- erties, and shows how it can be applied to solve real-world mathematical challenges more effectively.

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